Optical Communication 2008 E4.06 ISE 4.36 S09 A09 1/6

SOLUTIONS

1.

a) *A slab waveguide supports 3 TE modes, m = 0, 1 and 2. Which of these has the highest phase velocity, and why?*
Higher order modes are less strongly guided, so they have more light in the cladding and thus lower effective indices, & higher phase velocities. Answer is m=2.

b) *A transmitter launches 3 mW of optical power into a fibre 150 km long, and the optical power at the far end of the fibre is 100 nW. Calculate the propagation loss in dB/km.*
Loss is 10log(30,000) = 44.8 dB, so prop loss is 0.30 dB/km.

c) *An optical fibre is constructed having a silica core and an air cladding. Estimate its numerical aperture.*
$NA = \sqrt{(n_c^2 - n_o^2)}$; taking the core index as ≈ 1.5, and $n_o = 1$ gives NA = 1.12.

d) *An optical link is running at 2.5 Gbit/s. Estimate the thermal electrical noise assuming the receiver operates at room temperature.*
We can estimate the bandwidth as 1.25 GHz, and the electrical thermal noise spectral density is simply 4kT, so the noise power = $4kT\Delta f$ = 20 pW.

e) *Briefly describe one way in which a heterostructure can be used to increase the performance of a p-i-n photodiode.*
With a heterostructure a larger bandgap can be used in the upper p-layer, to allow enough layer thickness to minimise series resistance while avoiding photon absorption outside the depletion layer.

f) *If a particular semiconductor has an indirect bandgap, does this prevent, or significantly impede, its use to construct an optical detector?*
No, this is a major impediment to making a source but not significant for a detector.

g) *A photodetector operating at a nominal wavelength of 1.53 μm has a responsivity of 1.022 A/W. Calculate the quantum efficiency.*
Since $\mathcal{R} = \eta e\lambda/hc$, $\eta = hc\,\mathcal{R}/e\lambda$ = 6.63e-34 x 3e8 x 1.022 / (1.6e-19 x 1.53e-6) = 0.83.

h) *Two Fabry-Perot laser diodes have cavity lengths of 400 and 450 μm respectively. Which will have the closest spacing of longitudinal modes?*
The longer one.

i) *Briefly describe the difference between homodyne and heterodyne coherent receivers.*
Have a local oscillator with a frequency the same as or different to that of the transmitter respectively.

j) *Many silica fibres have a peak in attenuation between the 1.3 and 1.5 μm communications wavelength bands. What is the cause of this peak?*
This relates to the OH ion impurity which results from water.

2. *On pg. 1 the eigenvalue equations are given for TE modes in a symmetric slab waveguide as shown in Fig. 2.1*

a) *Consider two such slab waveguides, for use at a free-space wavelength of 1.530 μm. Guide A has n_1 = 1.470 and n_2 = 1.450. Guide B has n_1 = 1.465 and n_2 = 1.460. For each of these guides, determine the maximum thickness d for which only a single TE mode is supported* [6]

The cut-off condition for single-modedness can be written as $d = \frac{1}{2}\lambda_o/NA$. Students may remember or derive this. $NA = \sqrt{(n_1^2 - n_2^2)}$. Thus d_A = 3.166 μm, d_B = 6.328 μm.

b) *For each of guide A and B, with the thickness d chosen as calculated in (a), determine the effective index of the m=0 mode, to 4 significant figures. Show your work* [10]

Introduce $Y = \kappa d/2$. $X = k_{1x}d/2$, then the even eigenmode equation can be written as $Y = X\tan X$. Phase matching at the boundaries ($k_{1z}=k_{2z}$) gives us the relation $k_{1x}^2 + \kappa^2 = R^2$, with $R = NAk_o$. This we can write as $Y^2 + X^2 = (Rd/2)^2$. We are operating at the cutoff of m=1, and the cutoff condition from (a) can be re-written as $k_od = \pi/NA$. Thus $Rd/2 = NAk_od/2 = \pi/2$. Thus $Y^2 + X^2 = (\pi/2)^2$, and combining with the eigenvalue equation gives $X^2(1 + \tan^2X) = (\pi/2)^2$, or $X = \pm(\pi/2)\cos X$. We can estimate X = 1 as a crude starting value (by sketching the Y-X curve), and by successive approximation find X = 0.934.

Now we have $k_{1x} = 2X/d$, and $n'^2 = n_1^2 - (k_{1x}/k_o)^2 = n_1^2 - (2X/k_od)^2$, and using $k_od = \pi/NA$, $n'^2 = n_1^2 - (2XNA/\pi)^2$, or $n'^2 = n_1^2 - 0.354NA^2$. Inserting the corresponding indices gives n'_A = 1.4630, n'_B = 1.4632.

c) *Discuss the factors and trade-offs involved in choosing the core diameter and refractive index difference for a single mode optical fibre.* [4]

The fibre cutoff condition $r_o < 0.383\lambda/NA$ sets a requirement on the product of NA and core radius (or diameter) just as for the slab, so we can choose a larger core by using a smaller NA. Increasing the core size makes alignment to sources or between two fibres easier, but at the cost of lowering the NA, which produces (as above) an effective index closer to the cladding index. This means the mode is more weakly guided, and so is more susceptible to leakage due to bending (or to small variations in index or other defects). Increasing the NA to get stronger confinement, besides making alignment more difficult, is also limited by the difficulty of achieving large variations in the index of doped silica.

3. a) *To construct a certain optical link, two receivers are available. Receiver A has a detector quantum efficiency η of 0.8 and a noise equivalent power (NEP) of 12 pW/√Hz, while receiver B has $\eta = 0.7$ and NEP = 9 pW/√Hz. Assuming the link capacity (maximum bit rate) is limited by receiver noise, which receiver will provide the highest link capacity? Show your reasoning.* [4]

For receiver noise, SNR is proportional to received power over NEP, and received power is proportional to quantum efficiency. Therefore the receiver with the highest η/NEP value gives the best performance. This is receiver B (0.7/9 > 0.8/12).

b) *For the two receivers described in (a), which will provide the highest capacity if shot noise is the limiting factor?* [4]

For a shot noise limit, NEP is irrelevant, so Receiver A with the higher η is better.

c) *Show that for an optical receiver with quantum efficiency $\eta = 1$, the shot noise and receiver noise will be equal if twice the received optical power equals the noise equivalent power squared divided by the photon energy.* [4]

In photocurrent terms the receiver noise spectral density is $\mathcal{R}$(NEP) with $\mathcal{R}$ the responsivity. For $\eta = 1$, $\mathcal{R} = e/E_{ph}$, with E_{ph} the photon energy. The shot noise spectral density = $\sqrt{(2eI_{ph})} = \sqrt{(2e\mathcal{R}\Phi_R)}$, with Φ_R the received optical power. Equating the two gives (NEP)$e/E_{ph} = \sqrt{(2e^2\Phi_R/E_{ph})}$, which after some manipulation gives:
$\Phi_R = NEP^2/2E_{ph}$ as required.

d) *For a certain optical link whose maximum bit rate of 1.5 GBit/s is limited by shot noise, if we increase the required electrical SNR by 3 dB, what will the maximum bit rate now be?* [2]

This is a doubling of the electrical SNR, or an increase of the optical SNR by √2. Since SNR_{opt} goes as the inverse of √B, this requires a <u>halving</u> of the bit rate.

e) *Show that if an optical link is limited by receiver noise, the maximum bit-rate×length product B×L will be obtained for a length $L=1/2\alpha$, with α the attenuation coefficient. Calculate a corresponding bit rate, taking reasonable values for the relevant parameters. Explain why such a bit rate cannot be exploited in practice.* [6]

For receiver noise we can derive from the SNR equation: $BL = 2\left[\frac{\Phi_T}{NEP \cdot SNR}\right]^2 Le^{-2\alpha L}$

Setting d(BL)/dL=0 gives L=1/2α. Then $B = 2\left[\frac{\Phi_T}{NEP \cdot SNR}\right]^2 e^{-2}$. If we take SNR=12, NEP = 10 pW/√Hz, and Φ_T = 10 mW, we get: B ≈ 2×10^{15} bit/s. This is unachievable as it is far above the speed of any detector, source modulation or electronics generally.

4. a) *A light emitting diode (LED) has a horizontal active region located 4 μm from the flat, horizontal semiconductor-air interface. Calculate the maximum angle from the vertical, θ_m, for which emitted photons are able to cross the semiconductor-air interface, and find the fraction of photons which are emitted within this angular range. Assume that the active region emits photons equally in all directions. The refractive index of the semiconductor is 3.6.* [6]

The maximum angle is simply the critical angle for total internal reflection, which = $\sin^{-1}(n_2/n_1)$ – in this case $\sin^{-1}(1/3.6) = 16.1^\circ = 0.28$ rad. The solid angle subtended by $d\theta$ is $\sin\theta d\theta$, so the fraction f of light within θ_m is given by $\int_0^{\theta_m} \sin\theta d\theta \Big/ \int_0^{\pi} \sin\theta d\theta$,

giving $f = \dfrac{1-\cos\theta_m}{2}$, and using $\cos\theta \approx 1 - \theta^2$, so $f \approx \theta_m^2/4$, = 0.0196, or 2%.

b) *For the photons emitted within the angular range θ_m determined above, estimate the fraction absorbed between the active region and the surface. Assume an attenuation coefficient of 2×10^3 cm^{-1}. State any assumptions or approximations made.* [4]

Since we are only concerned with rays within 16° of vertical, the path length only varies between 4 and 4.2 , so we can neglect this variation and approximate the lost fraction as $(1 - \exp(-\alpha L)) = (1 - \exp(-2 \times 10^3 \times 4 \times 10^{-4}) = \underline{0.55}$.

c) *A uniform plastic layer of thickness 5 μm and refractive index 1.48 is now added to the semiconductor surface. Find the maximum emission angle θ_m which is now required for emitted photons to cross the semiconductor-plastic interface. For the photons crossing this interface, and neglecting absorption and Fresnel reflection at the interfaces, find the fraction which escape into the air. Briefly discuss the effect of this added layer on the overall external quantum efficiency η_{ext}.* [6]

Now we have $\theta_m = \sin^{-1}(1.48/3.6) = 24.3^\circ = 0.424$ rad, and $f \approx 0.424^2/4$, = 0.0449, or 4.5%. At the plastic-air boundary we have a second critical angle $\theta_{m2} = \sin^{-1}(1/1.48) = 42.5^\circ = 0.742$ rad. However, the light entering the plastic refracts so this corresponds to an angle in the semiconductor of $\sin^{-1}(\sin(42.5^\circ)\times 1.48/3.6) = 16.1^\circ$ as in part (a). The fraction of light entering the plastic that falls within the 2nd critical angle can therefore be calculated as $(1-\cos(16.1^\circ))/(1-\cos(24.3^\circ)) = 0.45$, or <u>45%</u>. Multiplying this by the 4.5% entering the plastic gives 2%, i.e. the same as in part (a). We would not expect a flat dielectric layer to reduce TIR losses, since the relation between the angles in the semiconductor and the air are unaffected by the presence of an intermediate layer.

d) *A domed plastic layer placed on the semiconductor surface can be used to increase η_{ext}. Why is such a structure unlikely to increase the amount of light that can be coupled from the LED into a single mode optical fibre?* [4]

Such a dome can increase the collimation of the output light, increasing the fraction within the numerical aperture of the fibre. However, it does this at the cost of an increase in effective area of the output beam, since the brightness of the source cannot be increased. This is only likely to be useful if the initial emission area is on the order than or smaller than the core of the fibre, but for SMF the core diameter is about 10 μm and the active area of an LED is almost certain to be much larger than this.

5. a) *Describe the structure and operating principles of erbium doped fibre amplifiers. Use diagrams where appropriate. Indicate the main performance criteria, and give typical values for these. Include a definition of amplified spontaneous emission (ASE) and discuss its significance.* [8]

Bookwork to be summarised from notes section 12.

b) *The ASE noise spectral density can be approximated as:*

$$(I_A{}^*)^2 = 4\frac{e^2\lambda}{hc}G(G\text{-}1)\Phi_o \qquad (5.1)$$

where G and Φ_o are the gain and input optical power respectively. Show that if a high gain amplifier is placed before the receiver, the resulting optical SNR is worse by a factor of √2 than without the amplifier, if the latter case is dominated by shot noise. State any assumptions or approximations used. Hence, deduce the rate at which SNR degrades with number of amplifiers in a particular link.

Assume quantum efficiency 1 in the detector, so the responsivity $R = e\lambda/hc$, and G>>1 so (G-1)≈ G. Then

$$(I_A{}^*)^2 = 4eRG^2\Phi_o$$

With the amplifier gain of G the photocurrent will be $RG\Phi_o$. With the amp, $SNR_{opt} = RG\Phi_o/(4eRG^2\Phi_o\Delta f)^{½}. = (R\Phi_o/4e\Delta f)^{½}$. For shot noise $(I_{sh}{}^*)^2 = 2eI_{ph}$, so without the amp, $SNR_{opt} = R\Phi_o/(2eI_{ph}\,\Delta f)^{½} = R\Phi_o/(2eR\Phi_o\Delta f)^{½} = (R\Phi_o/2e\Delta f)^{½}$, which is √2 less as stated. So electrical SNR degrades by 3 dB per amplifier.

Describe the most general conditions under which an optical amplifier will improve the SNR [12]

The SNR without the amplifier must be dominated by receiver (thermal and amplifier noise) to a sufficient extent.

6. *A silicon avalanche photodiode (Fig. 6.1) has intrinsic and avalanche layer thicknesses of w_i and w_A respectively, acceptor doping levels N_{A+}, N_{Ai} and N_{AA} in the p^+, intrinsic and avalanche layers respectively, and donor doping level N_D in the n layer.*

a) *Assuming that a sufficient bias voltage has been applied to fully deplete both the intrinsic and avalanche regions, derive expressions for the difference in electric field strength ΔE_i across the intrinsic region, and the difference in electric field strength*

ΔE_A across the avalanche region, in terms of the doping levels and layer thicknesses. Hence, derive an expression for the minimum bias voltage, V_{min}, required to fully deplete these two layers. [8]

In one dimension $E = (e/\varepsilon)\int \rho dx$, so for the fully depleted intrinsic layer $\Delta E_i = (e/\varepsilon)w_i N_{Ai}$. Similarly $\Delta E_A = (e/\varepsilon)w_A N_{AA}$. At V_{min}, the p+ layer is undepleted, so E=0 at the p-i boundary, so at the A-n boundary $E = \Delta E_i + \Delta E_A$. Charge balance gives $(w_i N_{Ai} + w_A N_{AA}) = w_n N_D$ with w_n the depleted thickness in the n layer. The voltage is now the area under the middle curve in the plot below, i.e. $V_{min} = \frac{1}{2} w_i \Delta E_i + w_A \Delta E_i + \frac{1}{2} w_A \Delta E_A + \frac{1}{2} w_n(\Delta E_A + \Delta E_i)$. Combining with the expressions above gives:

$$V_{\min} = \frac{e}{\varepsilon}\left[w_i N_{Ai}\left(w_A + \frac{w_i}{2}\right) + \frac{w_A N_{AA}}{2}\left(w_A + \frac{w_i N_{Ai} + w_A N_{AA}}{N_D}\right)\right]$$

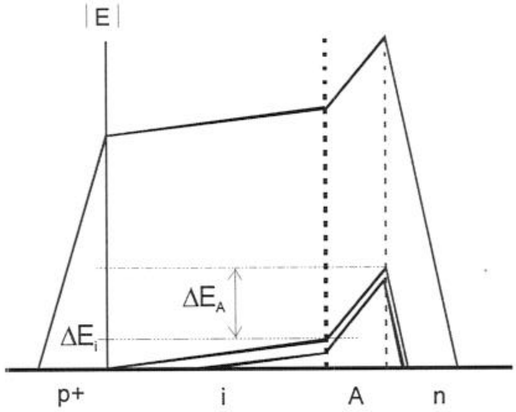
b) *Calculate ΔE_i, ΔE_A and V_{min} as derived in (a) for the following parameter values:*

$w_i = 8\ \mu m$, $w_A = 3\ \mu m$

$N_{A+} = 10^{21}\ m^{-3}$, $N_{AA} = 4\times10^{21}\ m^{-3}$, $N_{Ai} = 10^{20}\ m^{-3}$ and $N_D = 6\times10^{21}\ m^{-3}$

Plot and dimension the electric field profile E(x) for this case where the bias voltage equals V_{min}. On the same graph, sketch additional field distributions for bias voltages higher than and lower than this value. [8]

Inserting the values gives $\Delta E_i = 1.2 \times 10^6$ V/m, $\Delta E_A = 18 \times 10^6$ V/m, $V_{min} = 54.6$ V.



c) *Discuss the necessary and the desirable properties of the electric field distribution for such a device, and some factors limiting the optimisation of the field distribution.* [4]

Ideally the field should be at the level to give the saturation drift velocity throughout the depleted region, but not high enough to cause breakdown, except in the avalanche region, where it should be just above the breakdown level. In practice the field cannot be at the saturation level through most of the p+ level since it has to rise from zero, so we keep this layer thin by using a high doping level. Similarly the field has to rise linearly through the avalanche layer so cannot be uniform, and the n layer needs to be highly doped for the same reason as the p+.