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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE PART III/IV: MEng, BEng and ACGI

MICROWAVE TECHNOLOGY

Wednesday, 30 April 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	S. Lucyszyn
	Second Marker(s) :	C. Papavassiliou

Special instructions to candidates

Permeability of free space, $\mu_o = 4\pi \times 10^{-7}\ H/m$

Permittivity of free space, $\varepsilon_o \approx 8.854\ pF/m$

The Questions

1. An ideal air-filled rectangular waveguide has a guided-wavelength given by the following expression:

$$\lambda_g = \frac{\lambda_o}{\sqrt{1-\left(\frac{f_c}{f_o}\right)^2}} \qquad (1.1)$$

All variables have their usual meaning.

In addition, for the TE_{101} mode, the unloaded Q-factor for an air-filled rectangular waveguide resonant cavity is given by the following expression:

$$Q_u\Big|_{TE101} \cong \frac{2\,Volume}{\delta_o\ Area} \qquad (1.2)$$

All variables have their usual meaning.

a) For a half-height waveguide (i.e. its height dimension 'b' is half that of the width dimension 'a':

   i) Using (1.1), derive an expression for the length of the cavity in terms of 'a' and the various frequency terms. [4]

   ii) Using (i), derive an expression for the internal volume of the cavity. [2]

   iii) Using (i), derive an expression for the internal area of the cavity. [3]

   iv) Using (1.2) and assuming that $f_o/f_c = \sqrt{2}$, derive an expression for the unloaded-Q-factor in terms of 'a' and classical skin depth. [4]

   v) Using (iv), calculate the unloaded-Q-factor for a 15.5 GHz resonant cavity made with copper walls having a DC bulk conductivity of 5.8 x 10^7 S/m. [4]

b) For a cubical cavity (i.e. all internal dimensions are equal) derive an expression for the unloaded-Q-factor in terms of 'a' and classical skin depth and show that this has a 33.333% higher unloaded Q-factor. [3]

2. An 8 kΩ.cm high resistivity silicon (HRS) wafer has a measured dielectric constant of 12.86 and loss tangent of 6 x 10^{-4} at 300 GHz.

a) From the 300 GHz measurements, determine the effective conductivity.

[5]

b) Using the result from (a), how does the real part of conductivity differ from the original value quoted for the high resistivity silicon.

[3]

c) When a plane electromagnetic wave in free space has normal incidence to the HRS, determine a simple expression for the voltage wave reflection coefficient and from this calculate the reflectivity. Hint: state any simplifying assumption made for the dielectric property of the HRS.

[5]

d) If the HRS is heavily doped, such that its surface layer behaves like a good conductor, determine an expression for reflectivity, in terms of surface resistance and intrinsic impedance free space, given that a plane wave in free space has normal incidence. Hint, the total magnetic field has both forward and backward travelling waves, represented by the following expression:

$$H(z) = H(0)e^{-\gamma z} + H(0)e^{+\gamma z} \qquad (2.1)$$

All variables have their usual meaning.

[5]

e) Calculate the reflectivity in (d), given a surface impedance of 0.1 Ω.

[2]

3. Consider a straight cylindrical wire, having an outer radius 'R', length 'l' and bulk direct current (DC) conductivity 'σ_o'.

a) Write an equation for the DC resistance per unit length in terms of its conductivity and radius.

[2]

b) A magnetic flux exists within the wire when current 'I' is flowing, resulting in an internal inductance. Show that the internal DC inductance per unit length is independent of its radius. Hints:

Ampere's Law gives: $H = \frac{I}{2\pi R}\left(\frac{r}{R}\right)$ *where internal radius*, $r < R$ (3.1)

Magnetic Energy within the conductor: $W_m = \frac{\mu_o}{2} \int_{volume} H^2 \cdot dv$ *where* $v = (2\pi r)rl$ (3.2)

$$W_m \equiv \frac{1}{2} L_{DC} I^2 \qquad (3.3)$$

[4]

c) At high frequencies all the current will be found within one skin depth 'δ_o', at the surface of the wire. Give an expression for the high frequency (HF) impedance per unit length of the wire, in terms of 'σ_o' and 'δ_o'.

[2]

d) Using the results from (a) and (c), derive an expression for the HF resistance per unit length, in terms of DC resistance per unit length.

[2]

e) Using the results from (b) and (c), derive an expression for the HF inductance per unit length, in terms of DC inductance per unit length.

[2]

f) Since HF resistivity is inversely proportional to skin depth and HF inductance is directly proportional to skin depth, does this mean that the wire is purely resistive at microwave frequencies? Please briefly explain your answer.

[2]

g) Using the result in (e), prove that HF inductance is given by the following expression.

$$L_{HF} = \frac{\mu_o \delta_o}{2}\left(\frac{length}{width}\right) \qquad (3.4)$$

Hint: treat the wire as a hollow cylinder and roll it out into a rectangular slab.

[2]

h) Write down the well-know expression for skin depth within a conductor, in terms of frequency and bulk DC conductivity. Given a gold bond wire, having a bulk DC resistivity of 22.14 nΩ.m and 25 μm diameter, using equation (3.4), calculate the skin depth, the internal HF inductance per millimetre and HF resistance per millimetre at 3.6 GHz.

[4]

4. a) Describe, with the aid of a diagram, a branch-line coupler having a distributed-element implementation, indicating lengths and impedances. Indicate its main characteristics.

[5]

b) Replace the distributed-element implementation in 4(a) with an equivalent lumped-element version. How does the performance compare with that in 4(a)? With a coupler having an impedance of $Zo = 50\ \Omega$, calculate the components values for a design frequency of 1.85 GHz.

[5]

c) Replace the lumped-element implementation in 4(b) with an equivalent lumped-distributed version. With a coupler having an impedance of $Zo = 50\ \Omega$ and line sections of $\phi = 45°$, calculate the components values for a design frequency of 1.85 GHz.

[5]

d) Draw the topology of a balanced amplifier employing the coupler given in 4(a).

[5]

5. A short transmission line is used to transform an arbitrary termination impedance, $Z = R + jX$, to the reference impedance, Zo. The impedance looking into the transformer is given by the following expression:

$$z_{IN} = \frac{z + jz_{TX} \tan \vartheta}{z_{TX} + jz \tan \theta} \qquad (5.1)$$

where the normalised variables used have their usual meanings

a) Using this equation, derive the equations for the characteristic impedance of the transmission line, Z_{TX}, and the corresponding electrical length, ϑ.

[7]

b) From the expressions derived in 5(a), what are the mathematical limits for the resistive and reactive values of the termination impedance that can be mapped into the input impedance of the short transmission line transformer?

[3]

c) A load termination consisting of a 2 nH inductance in series with a 3 Ω resistance must be matched at 900 MHz to a 50 Ω reference impedance using a short transmission line transformer. Using expressions derived in 5(a) and 5(b), calculate Z_{TX} and ϑ for the short transmission line transformer.

[7]

d) Comment on the suitability, or otherwise, of implementing the short transmission line transformer calculated in 5(c) using conventional microstrip and thin-film microstrip technologies.

[3]

6. a) From first-principles, show that the time-average power flow along a lossless transmission line is independent of the line length and is equal to the incident wave power minus the reflected wave power at the termination impedance. Assume the characteristic impedance is purely real.

[5]

b) Define frequency dispersion in a transmission line. If a transmission lines is represented by a lumped-element model, show what happen to frequency dispersion when $RC = GL$.

[5]

c) Lossless transmission lines can be represented by an infinite number of lumped series-L/shunt-C sections. From first principles, derive a self-consistent equation for characteristic impedance, in terms of L and C, and use this to derive the corresponding bandwidth for this solution.

[5]

d) Calculate the input impedance, return loss and transmission loss at the cut-off frequency determined in 6(c).

[5]