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## The Questions

- 1 a) (i) Briefly describe breadth-first and depth-first search strategies.
- (ii) Briefly explain how the two different strategies can be implemented using the General Graph Search (GGS) algorithm.
- (iii) Using appropriate criteria for evaluating algorithms, compare and contrast depth-first and breadth-first search.
- (iv) Explain the relationship between Iterative Deepening depth-first search and depth-first/breadth-first search, and explain why it is an improvement even though it appears to involve more work.

[8]

- b) The Coloured Trails Problem is a problem to be solved on an  $M \times N$  grid of coloured squares.

A player in Coloured Trails has a set of chips, the same colours as the grid squares. A player may move into an adjacent square if it gives up a chip of the corresponding colour.

The problem is to find a path from a current grid location to a goal location.

Formulate, in Prolog or other declarative notation, the Coloured Trails Problem, such that it could be used with the General Graph Search (GGS) program.

- (i) Specify a representation for the grid.
- (ii) Specify a representation for the player state.
- (iii) Using an example, specify an initial state and a goal state.
- (iv) Specify the state change rules.

[12]

- 2 a) Define the terms *admissible heuristic* and *pathmax equation*, and explain why they are important in A\* search.

[4]

- b) A graph  $G$  is (explicitly) defined by a 3-tuple  $G = \langle N, E, R \rangle$ , where  $N$  is a set of nodes,  $E$  is a set of edges, and  $R$  is the incidence relation.

Give an implicit definition of a Graph  $G$ , in terms of nodes, the incidence relation, and paths. State what condition must be satisfied for the implicit definition to be equivalent to the explicit definition as a 3-tuple.

Explain how A\* searches the paths in the implicit definition.

[6]

- b) Explain why A\* search is optimal. Explain why one heuristic may still be “better” (more efficient) than another.

[6]

- d) Suppose there are two heuristics, *happy* and *gloomy*, that can be applied to a node in the search graph, given that for all nodes  $n$ ,  $gloomy(n) \leq happy(n)$ .

Suggest a modification to A\* search such that both heuristics could be used to guide the search.

[4]

- 3 a) Describe the Minimax Algorithm for two-player games.

[4]

- b) Explain the limitations of Minimax search, and describe an alternative algorithm for two-player games.

[6]

- c) Consider a game in which, for each round, there is a dice throw, followed by a move for MAX, followed by a move for MIN. The moves that MAX and MIN may make are dependent on the throw of the dice.

Explain how the algorithm of part (b) could be modified to play such *games of chance*. Illustrate your answer with a simple graph.

[6]

- d) Explain how Iterative Deepening Depth First could be used with the algorithm of part (b), and suggest why it might be advantageous.

[4]

- 4 a) Describe a procedure for converting a set of first-order formulas into clausal form, i.e. where every conjunct is of the form:

$$\neg a_1 \vee \neg a_2 \vee \dots \vee \neg a_m \vee b_1 \vee b_2 \vee \dots \vee b_n$$

[5]

- b) What is the relationship between such conjuncts and Prolog if:

(i)  $m = 0$  and  $n = 1$ .

(ii)  $m > 0$  and  $n = 1$ .

(iii)  $m > 0$  and  $n = 0$ .

[3]

- c) Consider the following English statements:

*One person is the ancestor of a second, if the first person is the parent of the second.*

*One person is the ancestor of a second, if the first person is the parent of a third person and the third person is the ancestor of the second.*

*Bob is the parent of Alice.*

*Alice is the parent of Eve.*

Express these statements as formulas of First Order Predicate Logic.

Transform these formulas into a set of Horn clauses, explaining the steps in the transformation.

Prove, using resolution and showing the unifiers, that *Bob* is the ancestor of *Eve*.

[8]

- d) Suppose the third and fourth statements in part (c) are replaced by the two statements:

*There is a person, and Bob is the parent of that person.*

*There is a person, and that person is the parent of Eve.*

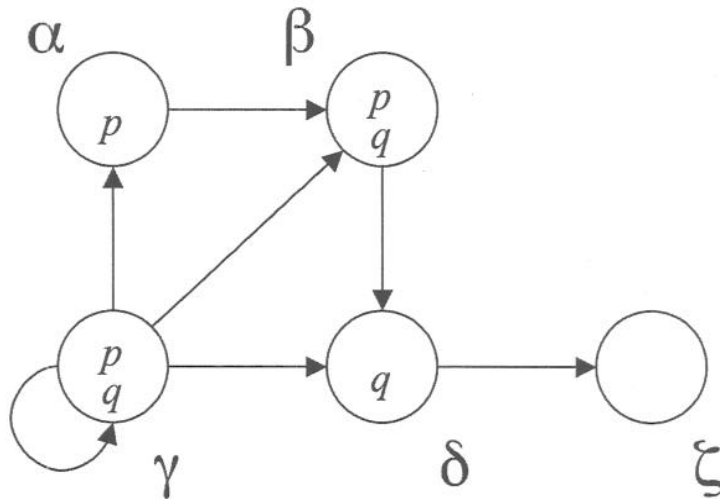
Explain (using a model and an inference rule for existential elimination) why it would now be unsafe to conclude *ancestor(Bob, Eve)*.

[4]

- 5 a) Discuss four properties for evaluating proof systems for propositional logic. [4]
- b) Explain the relationship between KE tableaux and disjunctive normal form. [4]
- c) A Search Engine is typically concerned with finding which pages satisfy certain criteria. Suppose an engine is required search for web pages containing all the terms  $a, b, \dots$ , at least one of the terms  $c, d, \dots$ , and none of the terms  $e, f, \dots$
- Describe how this search query could be represented as a Boolean formula. [3]
- d) Show how the KE proof procedure could be used to determine if a web page satisfies a query as formulated in part (c). Construct a (partial) KE tableau to illustrate your answer. State any additional assumptions you make. [9]

- 6 a) Describe how the syntax of propositional logic can be extended to give modal logic. [4]

- b) Consider the following diagram of a set of possible worlds.



Give the Kripke model represented by this diagram.

[4]

- c) Given the Kripke model  $M$  of part (b), say, with justification, whether each of the following formulas of modal logic are true or false:

- (i)  $M, \gamma \models \Box(p \vee q)$
- (ii)  $M, \beta \models \Box q \rightarrow \Diamond p$
- (iii)  $M, \alpha \models p \rightarrow \Diamond \Diamond q$
- (iv)  $M, \zeta \models \Box(p \wedge \neg p)$

[4]

- d) Show that the axiom schema B ( $P \rightarrow \Box \Diamond P$ ) does not hold in the class of all models. Show that the axiom schema B does hold in the class of all models which are symmetric.

[4]

- e) Using the KE tableau procedure for modal logic, show that:

$$\Diamond P \rightarrow \Box \Diamond P$$

is a theorem of S5. Annotate your proof to show which rules have been used.

[4]

