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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

MSc and EEE PART III/IV: MEng, BEng.and ACGI

Corrected Copy

OPTOELECTRONICS

Thursday, 1 May 2:30 pm

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho \, dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = - \iint_A \partial \underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial \underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial \underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial \underline{\mathbf{D}}/\partial t$

Material equations

$\underline{\mathbf{J}} = \sigma \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu \underline{\mathbf{H}}$

Vector calculus (Cartesian co-ordinates)

$\mathrm{grad}(\phi) = \partial\phi/\partial x \, \underline{i} + \partial\phi/\partial y \, \underline{j} + \partial\phi/\partial z \, \underline{k}$

$\mathrm{div}(\underline{\mathbf{F}}) = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\mathrm{curl}(\underline{\mathbf{F}}) = \underline{i} \, \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\} + \underline{j} \, \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\} + \underline{k} \, \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}$

$\mathrm{curl} \, \{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad} \, \{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2 \underline{\mathbf{F}}$

1. a) Prove that the tangential component of the electric field must match at a boundary between two media.

[4]

b) Show that the irradiance $\underline{S} = 1/2\ \mathrm{Re}\{\underline{E} \times \underline{H}^*\}$ represents the time average of the Poynting vector $\underline{\mathbf{S}} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$, if the fields $\underline{\mathbf{E}}$ and $\underline{\mathbf{H}}$ vary harmonically in time and E(x, y, z) and H(x, y, z) are their time-independent amplitudes.

[4]

c) A TE-polarized plane wave of wavelength λ_0 is incident from free space on a perfect metal surface at an angle θ as shown in Figure 1. Write down time-independent expressions for the incident and reflected waves, and find the reflected wave amplitude. Calculate the z-component of the irradiance and describe the interference pattern.

[12]

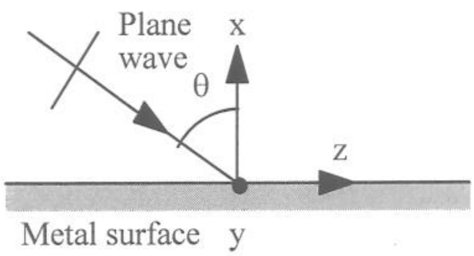

Figure 1.

2 a) Define the phase velocity v_{ph} and the group velocity v_g of a signal in terms of the angular frequency ω and the propagation constant k. Show how $1/v_{ph}$ and $1/v_g$ relate to the refractive index n of the medium in which the signal is travelling and to the frequency dependence of n. Express $1/v_g$ in terms of the wavelength dependence of n. Explain the difference between intermodal and intramodal dispersion.

[8]

b) A pulse of light is propagating in a step-index multi-mode fibre, which has core and cladding indices n_1 and n_2 respectively. Assuming that intermodal dispersion is the dominant form of dispersion, estimate the time spread of the pulse caused by propagation through a distance L. Hence, if a digital signal is coded using such pulses, estimate the product B x L, where B is the bit rate of the signal.

[4]

c) A pulse is propagating in a graded-index single-mode fibre, which has effective index n. Assuming that intramodal dispersion is the dominant form of dispersion, estimate the time spread of the pulse caused by propagation through a distance L. Hence, suggest the optimum region of the spectrum for minimum dispersion.

[8]

3. a) Stating all your assumptions, show that the time-independent scalar equation for TE modes in each layer of the slab waveguide in Figure 2 is $d^2E_T/dx^2 + \{n_i^2k_0^2 - \beta^2\}E_T = 0$, where $E_T(x)$ is the transverse field variation, $k_0 = 2\pi/\lambda$ is the free space wavenumber and β is the propagation constant.

[14]

b) Explain briefly how guided solutions are found, and sketch the lowest order transverse fields for a symmetric guide ($n_2 = n_3$).

[6]

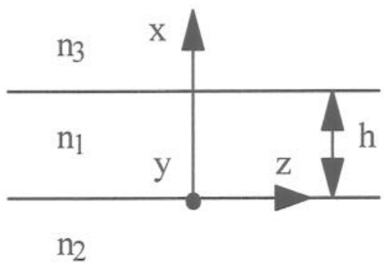

Figure 2.

4. a) The sandwich student hired to construct a Mach-Zehnder interferometric optical fibre temperature sensor has been fired after bungling the job. Their manager has found a sketch of the system in the student's logbook (Figure 3). How many cleaves and splices would allow a viable sensor to be constructed? What are their locations?

[4]

b) The student made their own couplers, measuring power outputs (in each case relative to the lower input) of 54% and 36% from ports A and B of coupler 1, and 55% and 30% from ports C and D of coupler 2. What are the insertion losses and coupling lengths of the two couplers? How do the latter results compare to the values needed?

[8]

c) Assuming unity input to (say) the lower port of coupler 1, derive expressions for the two outputs of the reconstructed sensor. What are the maximum and minimum powers that the manager is likely to measure as the sensor sweeps through its range?

[8]

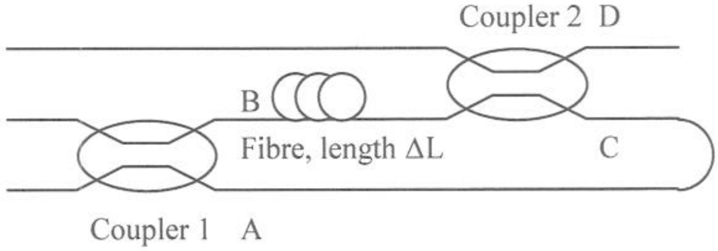

Figure 3.

5. Explain the importance of the following to modern telecommunications systems, illustrating your answer with examples where appropriate.

a) Direct gap materials

[5]

b) Epitaxial growth

[5]

c) Double heterostructure laser

[5]

d) PIN photodiode

[5]

6. The lumped-element rate equations for a semiconductor laser are:

$$dn/dt = I/ev - n/\tau_e - G\phi(n - n_0)$$
$$d\phi/dt = \beta n/\tau_{rr} + G\phi(n - n_0) - \phi/\tau_p$$

a) Explain the significance of the five terms I/ev, n/τ_e, $\beta n/\tau_{rr}$, $G\phi(n - n_0)$ and ϕ/τ_p. Why does τ_e appear in the upper equation and τ_{rr} in the lower one? How are τ_e and τ_{rr} related?

[6]

b) Explain how the photon lifetime τ_p is calculated in a Fabry-Perot laser with cavity length L and effective index n_{eff}.

[4]

c) An InGaAsP Fabry-Perot laser has a cavity length, width and depth of L = 250 μm, w = 3 μm and d = 0.1 μm and an effective index of $n_{eff} = 3.5$. Assuming that the electron lifetime is $\tau_e = 10^{-9}$ sec, the electron density at transparency is $n_0 = 10^{24}$ m^{-3} and the gain coefficient is $G = 10^{-12}$ m^3/sec, calculate the threshold current.

[10]