E3.07
AU1
ISE3.11

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2008

EEE/ISE PART III/IV: MEng, BEng and ACGI

Corrected Copy

DIGITAL SIGNAL PROCESSING

Friday, 2 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : P.A. Naylor
Second Marker(s) : D.P. Mandic

This page is intentionally left blank.

Digital Signal Processing

1. a) Consider an even symmetric sequence $x_e(n)$ and an odd symmetric sequence $x_o(n)$ such that

$$x_e(n) + x_o(n) = x(n) \tag{1.1}$$

where $x(n)$ is a given sequence. Write expressions for the sequences $x_e(n)$ and $x_o(n)$ that satisfy equation (1.1). [2]

b) The sequence $x(n)$ is a causal signal. By considering the cases $n < 0$, $n = 0$ and $n > 0$, determine the relationship between $x_e(n)$ and $x_o(n)$. [4]

c) Consider an N-point sequence $y(n) = y_R(n) + jy_I(n)$, where subscripts R and I indicate the real and imaginary parts respectively. The DFT of $y(n)$ is given by $Y(k)$. Derive expressions for $Y_R(k)$ and $Y_I(k)$. [4]

d) If $x(n)$ is real, causal and absolutely summable, find the relationships between $x_e(n)$ and $x_o(n)$ and between $X_R(k)$ and $X_I(k)$. Discuss this result. [4]

e) A digital audio recording of music with a sampling frequency of 48 kHz has been corrupted by intermittent sinusoidal tones at 23.2 kHz and 3.2 kHz. Show how you would use the DFT to detect when one or both of the sinusoidal tones are present in the signal. What factors may cause false alarms in your detection? [6]

2. a) State and describe the Noble Identities. [4]

b) Describe the general principles used to change the sampling frequency of a signal by a non-integer factor. [5]

c) Consider the two systems shown in Fig. 2.1 which represent the following operations for the upper and lower system respectively:

i) decimation by a factor of 3 followed by expansion by a factor of 2;

ii) expansion by a factor of 2 followed by decimation by a factor of 3.

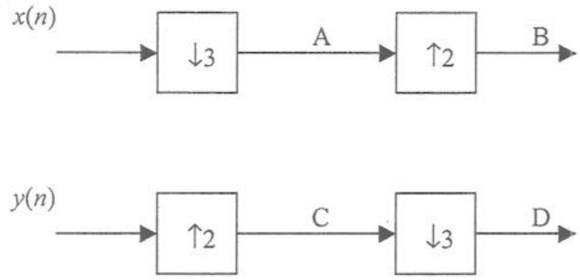

Figure 2.1

Denoting the input samples as $x(n) = \{x0, x1, x2, \ldots\}$ and $y(n) = \{y0, y1, y2, \ldots\}$, find the corresponding samples at the points in the Figure marked A, C and, hence, the first 6 samples at each of the points marked B and D. Comment on the importance of the order of operation of multirate processing blocks such as those in Fig. 2.1. [6]

d) The output of an L-fold expander is denoted $y(n)$. The input signal is $x(n)$ with frequency spectrum $X(e^{j\omega})$. Derive an expression for $Y(e^{j\omega})$. Explain in words the effect of the expander in the frequency domain and give an illustrative sketch of an example signal spectrum of your choosing. [5]

3. a) Write down brief descriptions of FIR and IIR filters. State the advantages and disadvantages of each. [6]

b) Consider an FIR filter with transfer function $B(z)$ and coefficients

$$b_k = \begin{cases} 1 & \text{if } 0 \leq k < 5, \\ 0 & \text{otherwise.} \end{cases}$$

Write out the system function of this filter. Hence, or otherwise, find the filter's difference equation. [6]

Derive a new filter with transfer function $G(z)$ as an IIR filter equivalent to $B(z)$ and write down its system function. Draw a labelled sketch of the magnitude of the frequency response of $G(z)$. [8]

4. Consider a discrete time filter with transfer function $H(z) = \frac{1}{2}\left(1 + z^{-1}\right)$.

a) Draw the signal flow graph of $H(z)$. [3]

b) Sketch the magnitude of the frequency response of $H(z)$ over the range of frequency ω from $\omega = 0$ to $\omega = 2\pi$. [3]

c) A comb filter, C(z), has a frequency response magnitude which exhibits spectral nulls and is also periodic with period $2\pi/M$. By considering $H(f(z))$ in which $H(z)$ is transformed such that z is replaced by a function $f(z)$, determine how $H(z)$ can be transformed into the comb filter $C(z)$. [11]

d) Draw the signal flow graph of $C(z)$. [3]

5. a) Give the definition of the z-transform of a discrete-time signal $x(n)$. What is meant by the *region of convergence* in the context of the z-transform? [2]

b) Find the z-transform and state the region of convergence for [6]

i) $x(n) = [1, -1, -9]$ where the time origin corresponds to the middle sample of $x(n)$;

ii) $y(n) = cos(\omega_0 n) \cdot u(n)$ where $u(n)$ is the unit step function.

c) Given a system defined as [6]

$$H(z) = \frac{1}{1 - 1.6z^{-1} + 0.65z^{-2}},$$

state the region of convergence for $H(z)$ to be causal and give an explanation as to whether the causal filter is also stable.

d) Write down the difference equation for $H(z)$ showing the relationship between the input samples and the output samples. Hence calculate the first 3 samples of the step response of the causal filter $H(z)$. [6]

6. a) For a signal $x(n)$, define the autocorrelation function, $\gamma_{xx}(l)$ and state the significant properties of $\gamma_{xx}(l)$. [5]

b) In room acoustics, sound waves that are generated in a reverberant room propagate from the source to a receiver both directly and by one or more reflections from walls and other reflective surfaces. In a simplified discrete-time model, assume that the signal at the receiver can be written

$$x(n) = h_0 s(n-k_0) + h_1 s(n-k_1) + h_2 s(n-k_2)$$

where $s(n)$ is the source signal, h_0, h_1 and h_2 are constant coefficients and k_0, k_1 and k_2 are integers.

Draw a labelled sketch of a room in plan view (from above) to illustrate this model and include an explanation of h_0, h_1, h_2, k_0, k_1 and k_2. [4]

State the principal ways in which this simplified model lacks accuracy. [3]

Derive an expression for the autocorrelation function of $x(n)$ in terms of the autocorrelation function of $s(n)$. [8]