

Answers COMMUNICATIONS II

EZ-4
2008

1. a) i) The mean $E[X(t)]$ of a wide-sense stationary random process doesn't depend on t , and the autocorrelation function $R_X(t_1, t_2)$ depends only on $\tau = t_1 - t_2$. [2, bookwork]
- The Wiener-Khinchine theorem says that the power spectral density is the Fourier transform of $R_X(\tau)$. [1, bookwork]
- ii) Give two statistically independent Gaussian random variables X and Y , [2, bookwork]

Rayleigh random variable $Z_1 = \sqrt{X^2 + Y^2}$ [2, bookwork]

Ricean random variable $Z_2 = \sqrt{(A+X)^2 + Y^2}$, A a constant

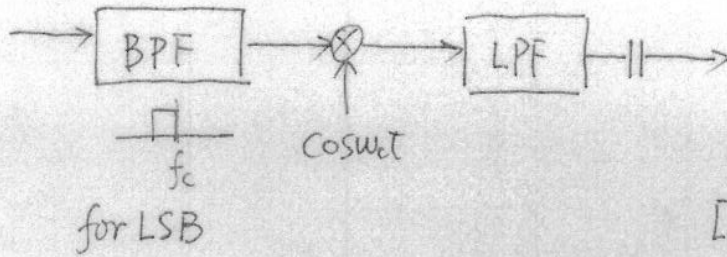
- iii) Ergodicity: A wide-sense stationary random process is ergodic if:

- its time average = ensemble average [2, bookwork]
- time autocorrelation function = ensemble autocorrelation function.

Yes, it is ergodic. [1, bookwork]

- b) i) • Synchronous detection: needs a local carrier that is synchronized with the incoming carrier. [1, bookwork]
- envelope detection: tracks the envelope of the signal; no local carrier needed. [1, bookwork]
 - coherent detection: the same as synchronous detection, a term used more often in digital communications. [1, bookwork]
 - Noncoherent detection: does not require phase synchronization at the receiver. [1, bookwork]

ii)



[3, bookwork]

iii) NO.

[3, bookwork]

In DPSK, the information symbols are differentially encoded, thus permitting differential detection.

c) i) Information of a symbol s : $I(s) = \log_2\left(\frac{1}{P}\right)$

Entropy of an information source $S = \{s_1, s_2, \dots, s_K\}$

$$H(S) = -\sum_{k=1}^K p_k \log_2(p_k) \quad \text{bits/symbol}$$

[5, bookwork]

ii) Mutual information

$$I(X; Y) = H(X) - H(X|Y)$$

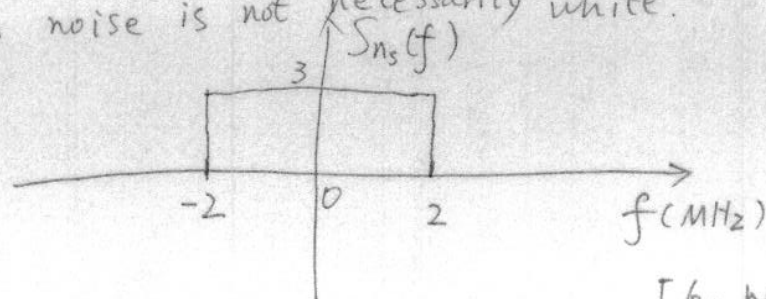
Channel capacity

$$C = \max_{P(X)} I(X; Y) \quad [5, bookwork]$$

Capacity formula $C = B \log_2\left(1 + \frac{S}{N}\right)$

d) i) "Additive white Gaussian noise" means noise is added to the signal, it has Gaussian distribution, and its power spectral density is a constant. [4, bookwork]
Gaussian noise is not necessarily white.

ii)

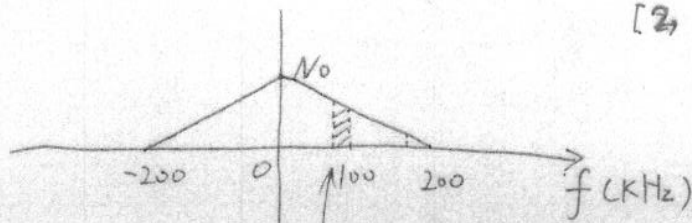


[6, new example]

2. a) i) Transmitted power $P_T = \frac{A^2 P}{4} = 2.5 \text{ W}$

Received power $P_R = \frac{2.5}{1000} = 2.5 \times 10^{-3} \text{ W} = 2.5 \text{ mW}$

[2, book work]



Noise power $P_N = \text{the area} \times 2$

$$= 2 \times \left(\frac{N_0}{2} \times 10 \text{ kHz} + \frac{0.05 N_0}{2} \times 10 \text{ kHz} \right)$$

$$= 10.5 \mu\text{W} \quad [3, \text{new example}]$$

$$\text{SNR} = \frac{P_R}{P_N} = \frac{2.5 \times 10^{-3}}{10.5 \times 10^{-6}} = 238 \quad (23.8 \text{ dB})$$

[3, new example]

ii) Noise power $P_N = 2 \times \frac{0.05 N_0}{2} \times 10 \text{ kHz}$

$$= 0.5 \times 10^{-6} \text{ W}$$

[3, new example]

$$= 0.5 \mu\text{W}$$

$$\text{SNR} = \frac{2.5 \times 10^{-3}}{0.5 \times 10^{-6}} = 5000 \quad (37 \text{ dB})$$

[3, new example]

b) i) 3dB bandwidth

$$|H_{de}(f_{3dB})| = \frac{1}{\sqrt{1 + (2\pi f_{3dB} RC)^2}} = \frac{1}{\sqrt{2}}$$

$$f_{3dB} = \frac{1}{2\pi RC}$$

equivalent bandwidth

$$B_{eq} = \frac{\int_0^\infty |H_{de}(f)|^2 df}{|H_{de}(0)|^2} = \int_0^\infty \frac{1}{1 + (2\pi f RC)^2} df$$

$$= \frac{1}{2\pi RC} \tan^{-1}(x) \Big|_0^\infty = \frac{\frac{\pi}{2}}{2\pi RC} = \frac{1}{4RC}$$

ii) After de-emphasis, noise PSD becomes

$$S_D(f) = \frac{f^2}{A^2} N_0 \frac{1}{1 + (f/f_{3dB})^2}$$

$$P_N = \int_{-W}^W S_D(f) df = \frac{N_0}{A^2} \int_{-W}^W \frac{f^2}{1 + (f/f_{3dB})^2} df$$

$$= \frac{N_0}{A^2} f_{3dB}^2 \int_{-W}^W \left[1 - \frac{1}{1 + (f/f_{3dB})^2} \right] df$$

$$= \frac{N_0}{A^2} f_{3dB}^2 \left[2W - 2f_{3dB} \tan^{-1} \left(\frac{W}{f_{3dB}} \right) \right]$$

$$= 2 \frac{N_0}{A^2} f_{3dB}^3 \left[\frac{W}{f_{3dB}} - \tan^{-1} \left(\frac{W}{f_{3dB}} \right) \right]$$

[6, new theory]

iii) Without de-emphasis

$$P_N = \frac{2N_0W^3}{3A^2}$$

[3, bookwork]

iv) Improvement

$$I = \frac{\frac{2N_0W^3}{3A^2}}{2 \frac{N_0}{A^2} f_{3dB}^2 \left[\frac{W}{f_{3dB}} - \tan^{-1} \left(\frac{W}{f_{3dB}} \right) \right]}$$

$$= \frac{W^3}{3 f_{3dB}^3 \left[\frac{W}{f_{3dB}} - \tan^{-1} \left(\frac{W}{f_{3dB}} \right) \right]}$$

$$RC = 6 \times 10^{-5} \Rightarrow f_{3dB} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 6 \times 10^{-5}} = 2.65 \text{ kHz}$$

$$I = \frac{15^3}{3 \times 2.65^3 \times \left[\frac{15}{2.65} - 1.4 \right]} = \frac{3375}{238} = 14.2$$

(11.5 dB)

[3, new application]

$$3. a) i) m(t) = A_m (\cos \omega_m t + \sin \omega_m t) \\ = \sqrt{2} A_m \cos(\omega_m t - \frac{\pi}{4})$$

$$P_s = \frac{1}{2} (\sqrt{2} A_m)^2 = A_m^2 \quad \Delta = \frac{2\sqrt{2} A_m}{2^n}$$

$$P_N = \frac{\Delta^2}{12} = \frac{8 A_m^2}{12 \times 2^{2n}} = \frac{2 A_m^2}{3 \times 2^{2n}}$$

$$SNR = \frac{P_s}{P_N} = \frac{3}{2} \times 2^{2n} \Rightarrow 6.02n + 1.76 \text{ dB}$$

[6, new example]

$$ii) 6.02n + 1.76 = 62$$

$$n \approx 10$$

[4, book work]

$$b) i) A/\sigma = 1/0.2 = 5$$

$$P_{e,ASK} = Q\left(\frac{A}{2\sigma}\right) = Q(2.5) \approx \frac{e^{-2.5^2/2}}{\sqrt{2\pi} \times 2.5} = 7 \times 10^{-3}$$

$$P_{e,FSK} = Q\left(\frac{A}{\sqrt{2}\sigma}\right) = Q\left(\frac{5}{\sqrt{2}}\right) = 2.1 \times 10^{-4}$$

$$P_{e,PSK} = Q\left(\frac{A}{\sigma}\right) = Q(5) = 3 \times 10^{-7}$$

[5, bookwork]

ii) Noncoherent detection

$$P_{e,ASK} = \frac{1}{2} e^{-\frac{A^2}{8\sigma^2}} = 2.2 \times 10^{-2}$$

$$P_{e,FSK} = \frac{1}{2} e^{-\frac{A^2}{4\sigma^2}} = 9.7 \times 10^{-4}$$

$$P_{e,PSK} = \frac{1}{2} e^{-\frac{A^2}{2\sigma^2}} = 1.9 \times 10^{-6}$$

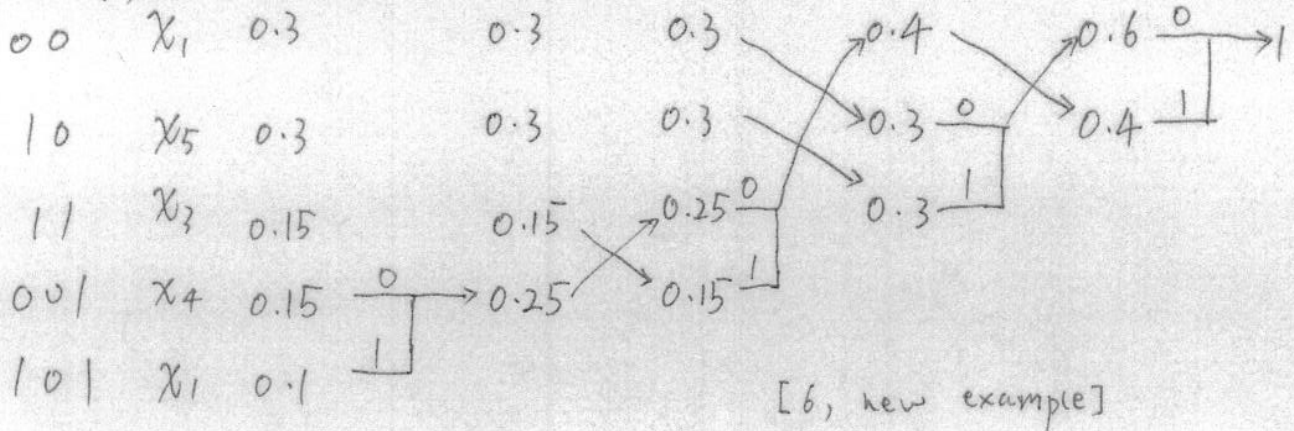
[5, bookwork]

$$\begin{aligned}
 \text{c) i)} \quad Q(x) &= \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta \\
 &\leq \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2}} d\theta \quad \text{since } \sin^2\theta \leq 1 \\
 &= \frac{\pi/2}{\pi} e^{-\frac{x^2}{2}} \\
 &= \frac{1}{2} e^{-\frac{x^2}{2}} \quad [4, \text{ new theory}]
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad Q(x) &= \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \\
 &= -\int_x^\infty \frac{1}{\sqrt{2\pi}t} de^{-t^2/2} \\
 &= -\frac{e^{-t^2/2}}{\sqrt{2\pi}t} \Big|_x^\infty - \int_x^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}t^2} dt \\
 &= \frac{e^{-x^2/2}}{\sqrt{2\pi}x} - \int_x^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}t^2} dt \\
 &\leq \frac{e^{-x^2/2}}{\sqrt{2\pi}x} \quad \text{Since the second term is non negative.}
 \end{aligned}$$

[6, new theory]

4. a) i)



ii) Average code word length

$$\bar{L} = 2 \times 0.3 + 2 \times 0.3 + 2 \times 0.15 + 3 \times 0.15 + 3 \times 0.1$$

$$= 2.25$$

Entropy

$$H(S) = - \sum p_i \log_2(p_i)$$

$$= 2 \times (-0.3 \times \log_2(0.3)) + 2 \times (-0.15 \times \log_2(0.15))$$

$$+ (-0.1 \times \log_2(0.1))$$

$$= 2.19$$

$$\eta = \frac{2.19}{2.25} = 97.3\% \quad [6, \text{new application}]$$

b) i)

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\text{row 4} + \text{row 3}} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \text{row 2 +} \\ \text{row 3} \end{array} \rightarrow \left[\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} \text{row 1 +} \\ \text{row 2 +} \\ \text{row 4} \end{array} \rightarrow \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right]$$

[6, new application]

Systematic form

ii) $g(z)h(z) = z^7 + 1$

$h(z) = z^4 + z^3 + z^2 + 1$

[4, new example]

$$\begin{array}{r} z^4 + z^3 + z^2 + 1 \\ \hline z^7 + 1 \\ z^7 + z^6 + z^4 \\ \hline z^6 + z^4 + 1 \\ z^6 + z^5 + z^3 \\ \hline z^5 + z^4 + z^3 + 1 \\ z^5 + z^4 + z^2 \\ \hline z^3 + z^2 + 1 \end{array}$$

iii) $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$

$d_{\min} = 3$ because the smallest number of dependent rows is 3.

error detection $t = d_{\min} - 1 = 2$ [4, bookwork]

error correction $t = \frac{d_{\min} - 1}{2} = 1$

iv) Hamming Bound $r = n - k \geq \log_2 \zeta(n, t)$

$\zeta(n, t) = \sum_{i=0}^t \binom{n}{i} \stackrel{t=1}{n=7} \binom{7}{0} + \binom{7}{1} = 8$

$\log_2 \zeta(n, t) = 3$

$r = n - k = 3$

∴ Yes, it's a 'perfect' code in this sense.

[4, new application]