

2008 -

①

Answers EE2 Semiconductor Devices (2008)

① a) Roles

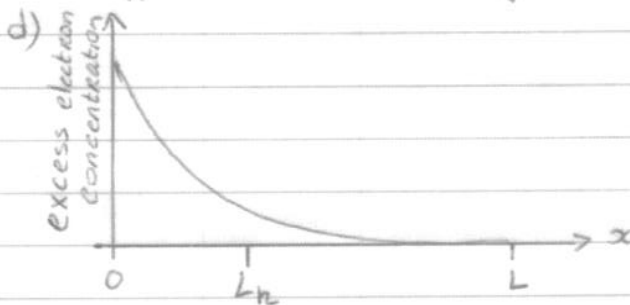
b)  $n_m = 5 \cdot 10^{17} \text{ cm}^{-3}$  majority carrier concentration

$$p_m = \frac{n_i^2}{N_D} = \frac{(1.45 \cdot 10^{10} \text{ cm}^{-3})^2}{5 \cdot 10^{17} \text{ cm}^{-3}} = 420.5 \text{ cm}^{-3}$$

= minority carrier concentration.

c) Drift is caused by an electric field

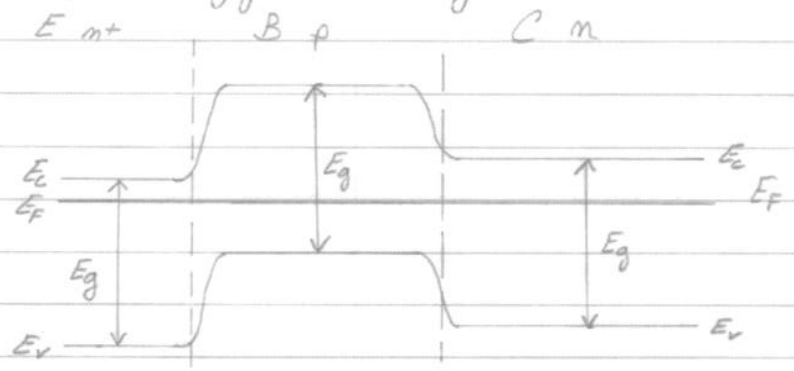
Diffusion is a result of carrier gradients



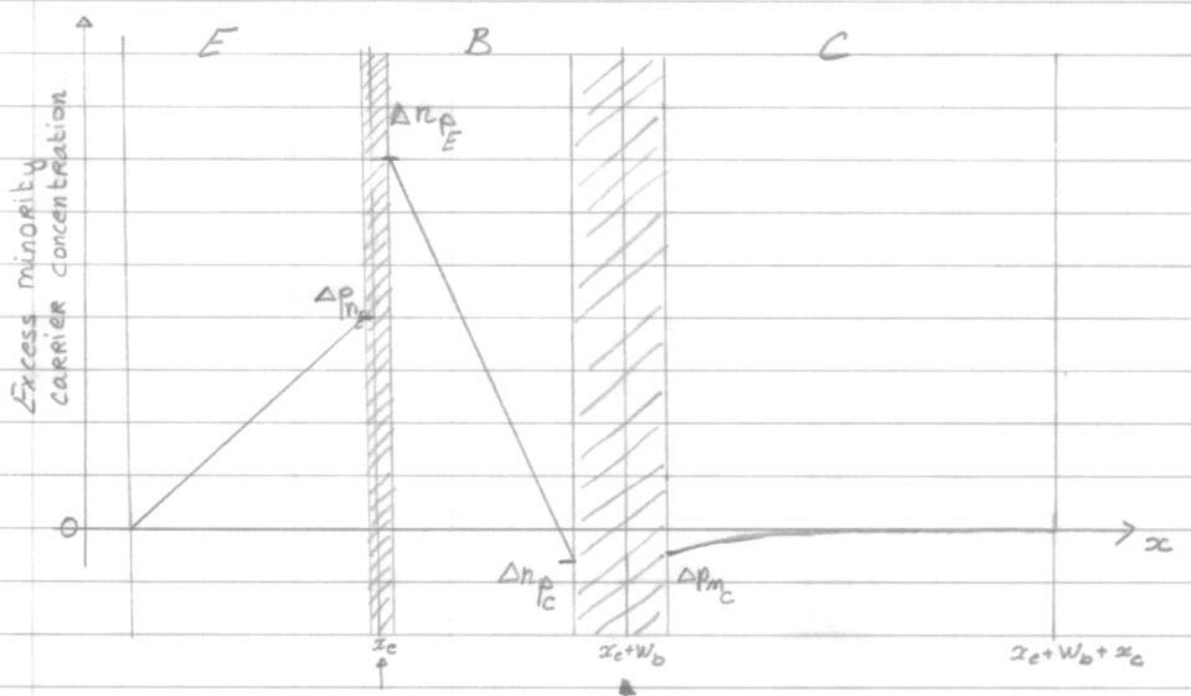
Should be an exponentially decreasing curve  
that goes to zero @  $x=L$ .

e) The main cause of delay in a BJT is the charge in the base region. This charge needs to be completely removed before the collector current can become zero.

2 a) Energy band diagram



b) Excess minority carrier concentration



$\Delta n_{pE} > \Delta n_{pC}$

$n_p'' = n_{p0} \exp\left(-\frac{x}{L_p}\right)$

$\Delta n_p = n_p'' - n_{p0} \approx -n_{p0} \frac{x}{L_p} = -\frac{n_i^2}{N_D} \frac{x}{L_p}$

$\Delta n_p \approx -n_{p0} = -\frac{n_i^2}{N_D}$

$\left. \begin{matrix} n_{p0} > p_{n0} \\ \Delta n_p < -p_{n0} \end{matrix} \right\} \Rightarrow -n_{p0} < -p_{n0}$

- Notes:
- Linear variation in E & B
  - Exponential variation in C
  - EB depletion width small
  - BC depletion width large

c) No recombination in the base  $\Rightarrow$   
 base current is the resupply of holes that are injected into the emitter  
 collector current is the electron component of the base-emitter current.

Thus:

$$I_B = \frac{\Delta p_{rE}}{x_e} e D_p$$

$$= (p'_{nE} - p_{rE0}) \frac{e D_p}{x_e}$$

$p'_{nE} \gg p_{nE0}$

$$\approx p'_{nE} e \frac{D_p}{x_e}$$

$$= e \frac{p_{nE0} D_p}{x_e} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$= e \frac{n_i^2 D_p}{x_e N_{DE}} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$I_C = - \left( \frac{\Delta n_{pE} - \Delta n_{pC}}{W_b} \right) e D_{nB}$$

$|\Delta n_{pE}| \gg |\Delta n_{pC}|$

$$\approx - \frac{\Delta n_{pE}}{W_b} e D_{nB}$$

$$= - (n'_{pB} - n_{p0B}) \frac{e D_{nB}}{W_b}$$

$n'_{pB} \gg n_{p0B}$

$$\approx - e \frac{D_{nB} n'_{pB}}{W_b}$$

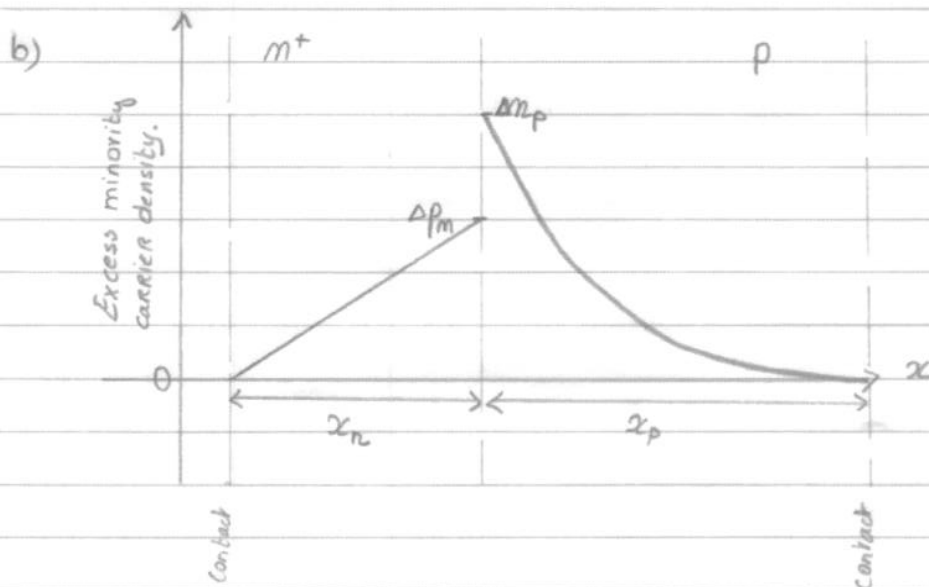
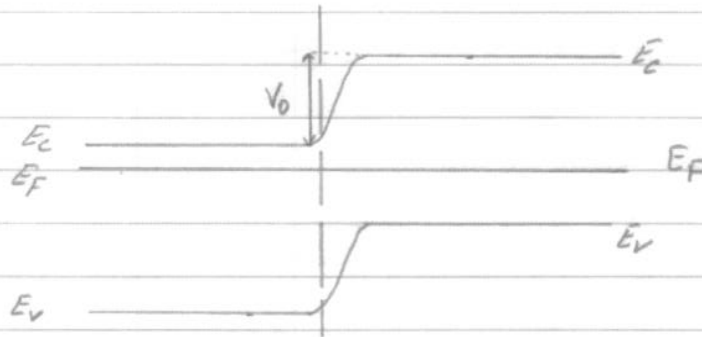
$$= - e \frac{D_{nB} n_{p0B}}{W_b} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$= - e \frac{D_{nB} n_i^2}{W_b N_{AB}} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$\begin{aligned}
 d) \quad \beta &= \frac{|I_C|}{I_B} \\
 &= \frac{e D_{nB} n_i^2 \exp\left(\frac{V_{EB}}{V_T}\right)}{W_b N_{AB}} \\
 &\quad \frac{e D_{pE} n_i^2 \exp\left(\frac{V_{EB}}{V_T}\right)}{x_e N_{DE}} \\
 &= \frac{D_{nB} N_{DE} x_e}{D_{pE} N_{AB} W_b} \\
 &= \frac{20 \frac{\text{cm}^2}{\text{s}} \cdot 10^{18} \text{cm}^{-3} \cdot 200 \cdot 10^{-7} \text{cm}}{12 \frac{\text{cm}^2}{\text{s}} \cdot 10^{16} \text{cm}^{-3} \cdot 200 \cdot 10^{-7} \text{cm}} \\
 &= 1.67 \cdot 10^3
 \end{aligned}$$

e) Spice uses the Ebers-Moll equations because they are valid for operation in both the saturation and active regions of operation - and therefore separate sets of equations for the different regions are not required. In addition, these equations only require the use of other existing SPICE models - the current controlled source and the pn diode.

3) a) Energy band diagram



Note: • variation in  $n^+$  layer linear  
• variation in  $p$  layer exponential.

$$\begin{aligned}
 Q_p &= \frac{e \cdot \Delta p_m \cdot x_n \cdot A}{2} = \frac{e (p_m' - p_{m0}) \cdot x_n \cdot A}{2} = \frac{A e x_n p_{n0}}{2} \left( \exp\left(\frac{V_{0B}}{V_T}\right) - 1 \right) \\
 &= \frac{e x_n n_i^2}{2 N_D} \left( \exp\left(\frac{V_{0B}}{V_T}\right) - 1 \right) \cdot A
 \end{aligned}$$

$$\begin{aligned}
 Q_n &= -e A \int_0^{x_p} \Delta p_p \exp\left(-\frac{x}{L_n}\right) dx \\
 &= e \cdot A \cdot \Delta p_p L_n \exp\left(-\frac{x}{L_n}\right) \Big|_0^{x_p}
 \end{aligned}$$

6

$$Q_n \approx +e A (n_p - n_{p0}) L_n (0 - 1)$$

$$= -\frac{e A n_i^2 L_n}{N_A} \left( \exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right)$$

d) Since  $n^+p$  junction  $I_{tot} \approx I_n$

$$I_n = \frac{Q_n}{\tau_n} \quad \text{Recombination current.}$$

$$I_n = \frac{-e A \Delta n_p L_n}{\tau_n}$$

$$J_n = \frac{I_n}{A} = \frac{-1.6 \cdot 10^{-19} \text{ C} \cdot 2 \cdot 10^{12} \text{ cm}^{-3} \cdot 500 \cdot 10^{-7} \text{ cm}}{0.1 \cdot 10^{-6} \text{ s}}$$
$$= -16 \cdot 10^{-5} \frac{\text{A}}{\text{cm}^2}$$

e) i)  $I = 0 \text{ A}$  open circuit.

ii)  $V_d > 0 \text{ V}$  forward bias voltage remains until excess charge is removed.

2E Electromagnetic Fields 2008 – Solutions

4. a) Key contributions were as follows:

i) Wilhelm Roentgen: Discovered X-rays

Heinrich Hertz: Discovered radio waves

[2]

ii) Alexander Graham Bell: Invented the telephone

Guglielmo Marconi: Invented the radio

John Logie Baird: Invented the television

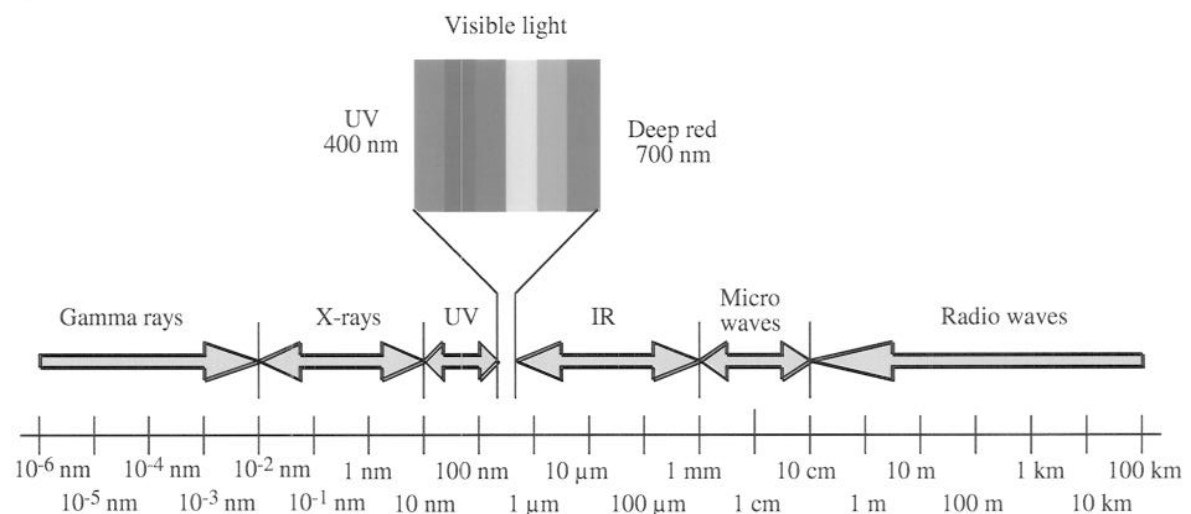
[3]

iii) John Tyndall: Demonstrated light guidance by total internal reflection in a water jet

Charles Kao and George Hockham: Proposed low loss optical fibre communications

[2]

b)



[1 mark for each band = 7]

c) Radio waves – main difficulty is diffraction; overcome using phased arrays

[2]

Microwaves – main difficulty is attenuation; overcome by avoiding water absorption frequency.

[2]

Light waves – main difficulty is scattering; overcome by confinement inside an optical fibre.

[2]

5. a) The phase velocity is the velocity of a single travelling wave.

The group velocity is the velocity of a group of waves, which can represent a modulated carrier and hence describe information propagation.

[3]

The two constituent waves with frequencies  $\omega + \Delta\omega$  and  $\omega - \Delta\omega$  must have corresponding propagation constants  $k + \Delta k$  and  $k - \Delta k$ . The combined signal can therefore be written as:

$$A(z, t) = A_0 \{ \exp\{j[(\omega + \Delta\omega)t - (k + \Delta k)z]\} + \exp\{j[(\omega - \Delta\omega)t - (k - \Delta k)z]\} \} \text{ or}$$

$$A(z, t) = A_0 \{ \exp[j(\Delta\omega t - \Delta k z)] + \exp[-j(\Delta\omega t - \Delta k z)] \} \exp[j(\omega t - kz)] \text{ or}$$

$$A(z, t) = 2A_0 \cos(\Delta\omega t - \Delta k z) \exp[j(\omega t - kz)]$$

This result describes a carrier with a modulating envelope in the form of a travelling sinusoidal wave. The velocity of the envelope is  $v_g = \Delta\omega/\Delta k$ .

[3]

b) If the dispersion characteristic of the ionosphere is  $\omega = \sqrt{[\omega_p^2 + c^2 k^2]}$ , then:

$$v_{ph} = \omega/k = \sqrt{[(\omega_p^2/k^2) + c^2]} \text{ and}$$

[3]

$$v_g = d\omega/dk = c^2/\sqrt{[(\omega_p^2/k^2) + c^2]}$$

[3]

For the atmosphere, which has  $\omega_p = 0$ ,  $\omega = ck$ , so:

$$v_{ph} = \omega/k = c$$

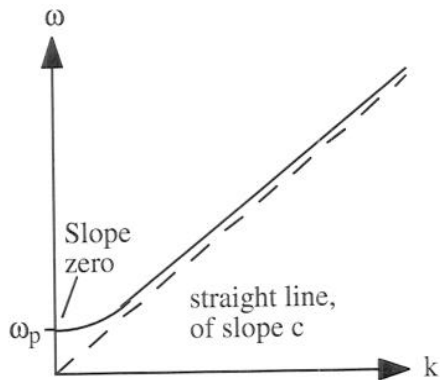
[3]

$$v_g = d\omega/dk = c$$

[3]

c) The dispersion diagram is as shown below. Since  $v_g$  tends to zero when  $\omega$  tends to  $\omega_p$ , there can be no transmission of information in this frequency range.

[3]



[3]

d) Rewriting the equation for the dispersion characteristic, we get:  $\omega^2 = \omega_p^2 + c^2 k^2$

Hence  $k = (1/c) \sqrt{\omega^2 - \omega_p^2}$ .

If  $\omega < \omega_p$ , this result can be written as  $k = \pm j(1/c) \sqrt{(\omega_p^2 - \omega^2)} = \pm jk'$

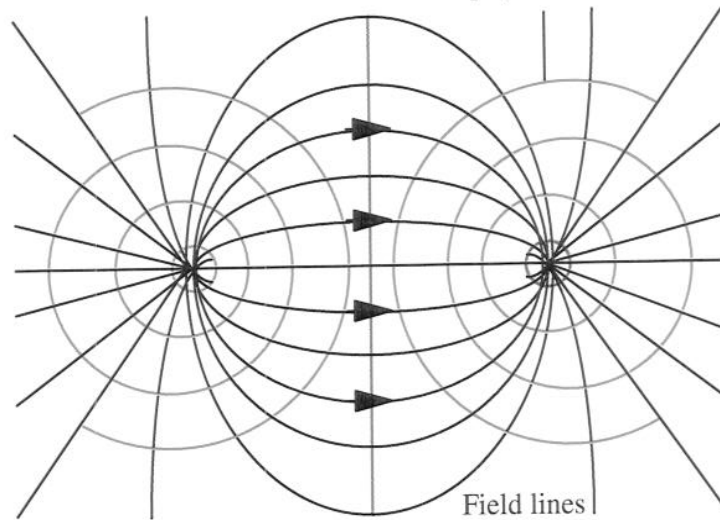
[3]

A wave solution  $A(z, t) = A_0 \exp[j(\omega t - kz)]$  then becomes  $A(z, t) = A_0 \exp(j\omega t) \exp(\pm k'z)$ , i.e. an exponentially decaying wave.

The significance of this result is that waves with angular frequencies less than  $\omega_p$  will not be able to penetrate the ionosphere; instead, they will be reflected (as Marconi found).

[3]

6. a) Because  $a \ll d$ , the equipotentials are circles, centred approximately on  $x = \pm d/2$  (in a more detailed analysis, the centres depend on the radius). The field lines form an orthogonal set.



[9]

The electric flux density at a radius  $r$  from a cylindrical line charge can be found from Gauss' law as  $\underline{D} = q/2\pi r \underline{i}$

For the left-hand wire, the electric flux density at P is  $\underline{D}_1 = q/2\pi r \underline{i}$

For the right hand wire, the corresponding value is  $\underline{D}_2 = q/2\pi(d - r) \underline{i}$

The total electric flux density is therefore  $\underline{D} = (q/2\pi) \{1/r + 1/(d - r)\} \underline{i}$

[3]

The electric field is  $\underline{E} = (q/2\pi\epsilon_0) \{1/r + 1/(d - r)\} \underline{i}$ ,

The voltage between the wires is then  $V = \int_a^{d-a} E dr = \int_a^{d-a} (q/2\pi\epsilon_0) \{1/r + 1/(d - r)\} dr$ , or

$V = (q/2\pi\epsilon_0) [\log_e\{r/(d - r)\}]_a^{d-a} = (q/\pi\epsilon_0) \log_e\{(d - a)/a\}$

The capacitance per unit length is therefore  $C = q/V = \pi\epsilon_0/\log_e\{(d - a)/a\}$ .

[6]

b) The magnetic field at a radius  $r$  from a wire carrying a current  $I$  can be found from Ampere's law as  $\underline{H} = I/2\pi r \underline{\theta}$ .

For the left-hand wire, the magnetic field at P is  $\underline{H}_1 = I/2\pi r \underline{j}$

For the right-hand wire, the corresponding value P is  $\underline{H}_2 = I/2\pi(d - r) \underline{j}$

The total magnetic field is then  $\underline{H} = (I/2\pi)\{1/r + 1/(d - r)\} \underline{j}$

So the total flux density at P is  $\underline{B} = (\mu_0 I/2\pi) \{1/r + 1/(d - r)\} \underline{j}$

[3]

So the flux per unit length crossing between the two wires is

$\Phi = (\mu_0 I/2\pi) \int_a^{d-a} \{1/r + 1/(d - r)\} dr$

Integrating, we get

$\Phi = (\mu_0 I/2\pi) [\log_e\{r/(d - r)\}]_a^{d-a} = (\mu_0 I/\pi) \log_e\{(d - a)/a\}$

The inductance  $L = \Phi/I$  is then  $L = (\mu_0/\pi) \log_e\{(d - a)/a\}$

[6]

The phase velocity is  $v_{ph} = 1/(LC)^{1/2} = 1/(\mu_0\epsilon_0)^{1/2}$

[3]