IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE PART II: MEng, BEng and ACGI

Corrected Copy

ANALOGUE ELECTRONICS 2

Monday, 15 June 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Papavassiliou, C. Papavassiliou
Second Marker(s) : E. Rodriguez-Villegas, E. Rodriguez-Villegas

The Questions

1. (Compulsory)

a) Draw a small signal equivalent circuit for a BJT. Label all components. List the differences between the BJT and FET small signal equivalent circuit.

[4]

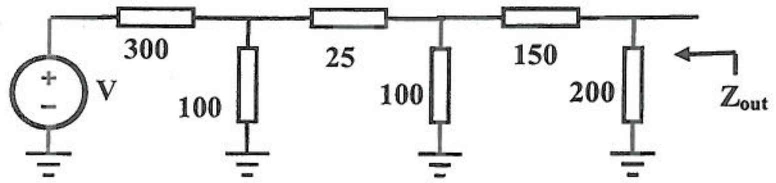
b) Write the two equations defining the transmission (ABCD) parameters in terms of the terminal voltages and currents of a two-port device. Express the transmission parameters in terms of the four gains.

[4]

c) Calculate the input impedance of an inverting amplifier built using an "op-amp" whose open loop gain is 9, constant with frequency. The feedback resistor is $1\text{k}\Omega$ and the input resistor is $100\,\Omega$.

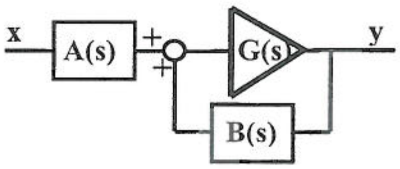
[4]

d) Calculate the Thevenin equivalent of the following network:



[4]

e) Calculate the transfer function of the following filter, if $A(s) = P(s)/Q(s)$, $B(s) = R(s)/Q(s)$.



[4]

f) Write an expression for the frequency dependence of the open loop gain of a typical, dominant pole, commercial op-amp. What is meant by the term "Gain-Bandwidth Product" ?

[4]

g) Identify the feedback connection type (series or shunt, at input and output) of the emitter degenerated common emitter amplifier. Calculate the transconductance of this circuit in terms of the transistor transconductance and the emitter degeneration resistance and the current gain β.

[4]

h) Explain how, by the use of feedback, a generic (not in any way ideal) voltage amplifier of finite gain-bandwidth product can be turned into an ideal transconductor at a single frequency. Identify the polarity, type of feedback connection and loop gain required to do this. What are the units of the reverse gain?

[4]

i) State the shunt version of the Miller theorem. How can shunt feedback be used to turn a transistor into a unilateral transconductor at a single frequency?

[4]

j) List the 3 types of single transistor amplifier. Comment on the magnitude of each amplifier's input and output impedance. Which one would you prefer to use for driving small impedance loads, and why?

[4]

2.

a) Derive the transfer function of the circuit in figure 2.1, assuming the op-amp is ideal.

[10]

b) What filter function does this circuit perform? Write expressions for the peak gain H_0, the natural frequency ω_n and the Quality factor Q. What is the maximum possible value of Q ?

[10]

c) How does the transfer function of this circuit change if the circuit is driven by a signal source of Thevenin resistance R_T ? Comment on any changes of the peak gain H_0, the natural frequency ω_n and the Quality factor Q.

[5]

d) Calculate the transfer function of this circuit if the op-amp has a finite gain G_0. Does the function of the filter change? Comment on any changes of the peak gain H_0, the natural frequency ω_n and the Quality factor Q. [5]

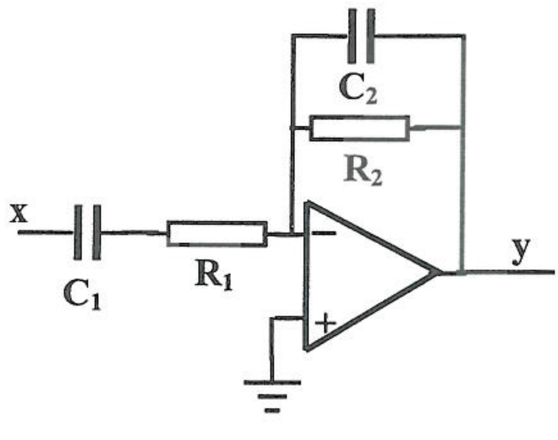


Figure 2.1

3.

a) Name the amplifier in figure 3.1 and calculate its DC voltage gain when driven by an ideal voltage source and driving an ideal voltmeter.

[5]

b) Draw a small signal AC equivalent circuit for this amplifier. Derive an expression for the frequency dependent input impedance of the amplifier.

[10]

c) Derive an expression for the output impedance of this amplifier if the input x is driven by a source of Thevenin impedance Z_T.

HINT: This is a feedback circuit!

[5]

d) Derive an expression for the frequency response of the voltage gain of this amplifier if x is driven by a Thevenin source with V_T , R_T. Write approximate expressions for the frequencies of the input and output poles of this amplifier. Explain why this amplifier will always have a larger bandwidth than a common Emitter amplifier with equal gain.

[10]

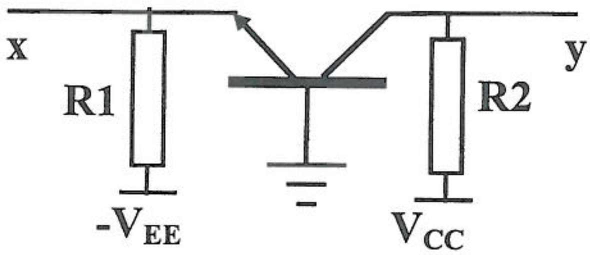


Figure 3.1

4.

a) Calculate the frequency dependent input impedance of the circuit within the dashed line box in Figure 4.1. [10]

b) Calculate the transfer function from x to y. What type of filter is this? Write expressions for the peak gain, natural frequency and quality factor of this filter. [10]

c) Calculate the transfer function from x to z. What type of filter is this? Write expressions for the peak gain, natural frequency and quality factor of this filter. [5]

d) Design a non-inverting second order low pass filter using the circuit in figure 4.1 and additional components. Draw a schematic diagram of the required additional circuit. [5]

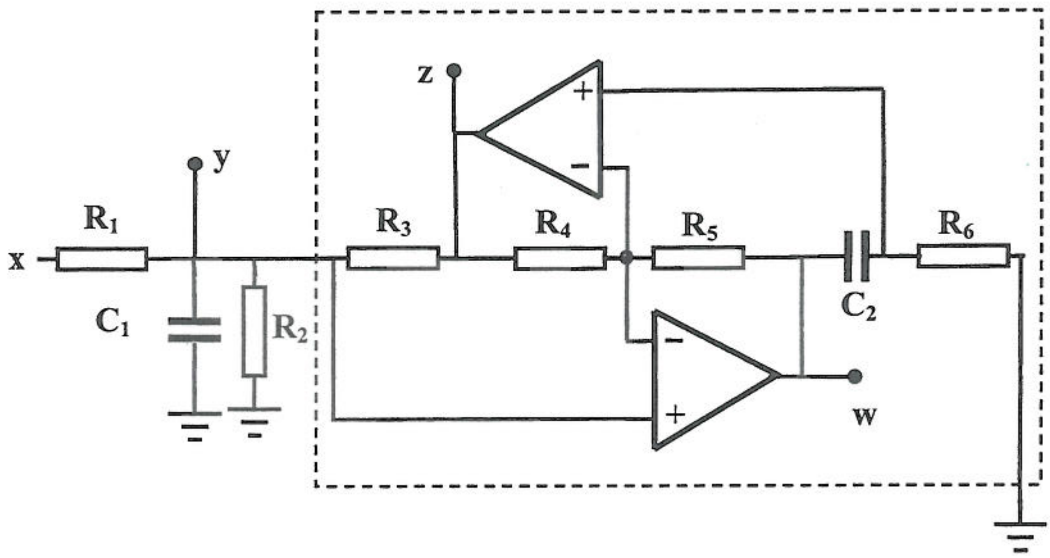


Figure 4.1

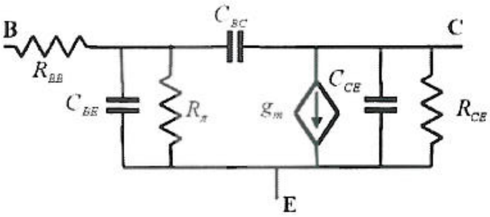
Analogue Electronics II

E22 ANSWERS 2008

The ANSWERS

1. Compulsory (4 points each) [a,b,e,f,g,h,I,j: taught, c,d: examples]

a)



The FET has the same model if $B \to G, C \to D, E \to S$, and omit R_π

b) $v_1 = Av_2 - Bi_2, i_1 = Cv_2 - Di_2$.
$A = 1/G_{12}, B = -1/Y_{21}, C = 1/Z_{21}, D = -1/H_{21}$

c) By the Miller theorem it is $100 + 1000/(1-G) = 100 + 1000/10 = 200\Omega$

d) Starting from left, there are 3 dividers:

$V_{T1} = V/4, R_{T1} = 75$ after adding 25Ω in series and 100Ω in parallel we get $V_{T2} = V_{T1}/2 = V/8, R_{T2} = 50$. Adding 150Ω in series and 200Ω in parallel we get $V_{T3} = V_{T2}/2 = V/18, R_{T3} = 100$.

e) $H = y/x = AG/(1-GB) = \dfrac{PG}{Q(1-GR/Q)} = \dfrac{PG}{Q-GR}$

f) $G(s) = \dfrac{G_0}{1+s\tau}$. The gain bandwidth product is $GBW = 2\pi G_0/\tau$

g) series-series. $g'_m = g_m/(1+g_m R_E)$

h) The key word is that the end result is ideal, so that input and output admittances must exactly vanish. Positive shunt-shunt feedback, with loop gain=1 is required. Shunt shunt feedback is applied on transimpedance amplifiers, so that the reverse path gain is a (trans) conductance.

i) The input admittance of a voltage amplifier of gain G with an admittance Y_F connecting its input and output equals the amplifier input admittance increased by (1-G) Y_F. Inductive Shunt miller feedback can be used to resonate with, and neutralis,e C_{BC} turning the transistor into a unilateral transconductor at the resonance frequency.

j) CE: medium Z_{in} ,high Z_{out}, CB: low Z_{in} ,high Z_{out}, CC: medium Z_{in} ,low Z_{out}. The common collector because of its low output impedance.

2. [new problem, computations related to a lab assignment]

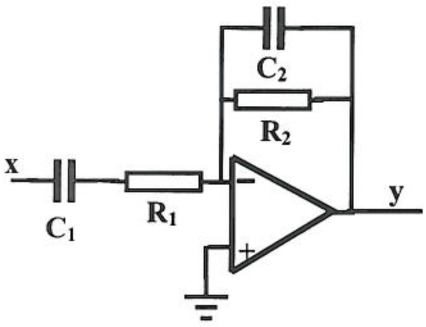
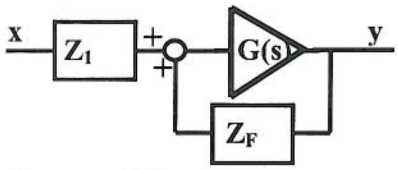


Figure 2.1

a) The circuit is



$$Z_1 = R_1 + \frac{1}{sC_1} = R_1 \frac{1+s\tau_1}{s\tau_1} \;,\; Z_F = \left(\frac{1}{R_2} + sC_2\right)^{-1} = \frac{R_2}{1+s\tau_2}$$

$$H_\infty = -Z_F / Z_1 = \frac{-R_2}{R_1} \frac{s\tau_1}{(1+s\tau_2)(1+s\tau_1)} = \frac{-R_2}{R_1} \frac{\tau_1}{\tau_1+\tau_2} \frac{(\tau_1+\tau_2)s}{s^2\tau_1\tau_2 + s(\tau_1+\tau_2)+1}$$

[10]

b) this is a band pass filter with

$$H_0 = \frac{-R_2}{R_1}\frac{\tau_1}{\tau_1+\tau_2} \;,\; \omega_n = \frac{1}{\sqrt{\tau_1\tau_2}},\; 2\zeta/\omega_n = \tau_1+\tau_2 \Rightarrow Q = \frac{1}{\sqrt{\xi}+\sqrt{1/\xi}} < \frac{1}{2} \;,\; \xi = \frac{\tau_1}{\tau_2}$$

[10]

c) R_T needs to be added to Z_1. In this case,

$$Z_1' = R_1 + \frac{1}{sC_1} + R_T = (R_1+R_T)\frac{(R_1+R_T)sC_1+1}{s(R_1+R_T)C_1} = R_S \frac{1+s\tau_3}{s\tau_3}$$

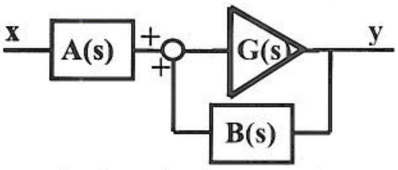
where $\tau_3 = (R_1+R_T)C_1$

$$H' = -Z_F / Z_1' = \frac{-R_2}{R_1}\frac{s\tau_3}{(1+s\tau_2)(1+s\tau_3)} = \frac{-R_2}{R_1}\frac{\tau_3}{\tau_3+\tau_2}\frac{(\tau_3+\tau_2)s}{s^2\tau_3\tau_2 + s(\tau_3+\tau_2)+1}$$

This is again a bandpass filter with a lower gain, lower natural frequency, and lower Q.

[5]

d) The circuit is of the form:



and its transfer function is $H = y/x = -AG_0/(1+G_0B)$

The block gains are

$$A = \frac{Z_F}{Z_1 + Z_F},\ B = \frac{Z_1}{Z_1 + Z_F}$$

Since we have calculated $H_\infty = \frac{-Z_F}{Z_1}$ it makes sense to divide by Z_1 so that:

$$A = \frac{H_\infty}{1+H_\infty},\ B = \frac{1}{1+H_\infty}$$

$$\Rightarrow H = -\frac{H_\infty G}{1 + H_\infty + G} = \frac{-H_0 \dfrac{2\zeta s/\omega_0 G_0}{s^2/\omega_0^2 + 2\zeta s/\omega_0 + 1}}{1 + H_0 \dfrac{2\zeta s/\omega_0}{s^2/\omega_0^2 + 2\zeta s/\omega_0 + 1} + G_0} =$$

$$= \frac{-H_0 2\zeta s G_0/\omega_0}{(1+G_0)\left(s^2/\omega_0^2 + 2\zeta s/\omega_0 + 1\right) + 2\zeta s H_0/\omega_0} = \frac{H_0 G_0}{(1 + G_0 + H_0)} \frac{2\zeta\left(\dfrac{1+G_0+H_0}{1+G_0}\right)s/\omega_0}{s^2/\omega_0^2 + 2\zeta s/\omega_0\left(\dfrac{1+G_0+H_0}{1+G_0}\right) + 1}$$

This is still a band pass filter, with the same natural frequency but reduced Q and gain.

[5]

3. [mix of taught theory and computation]

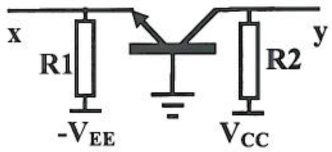
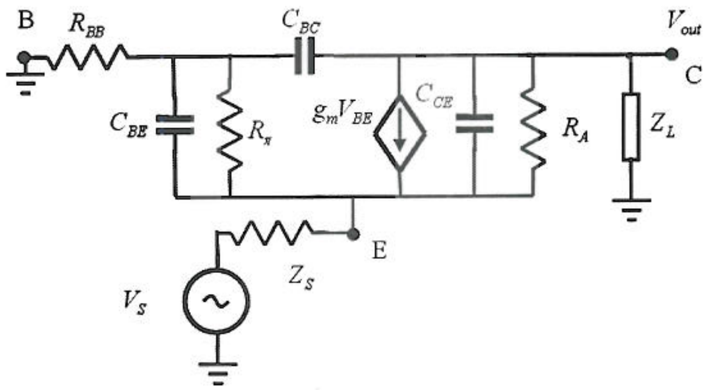


Figure 3.1

a) Common base. The gain is $G = g_m R_2 = i_C R_2 / V_T = \left(\left(V_{EE} - V_{BE}\right) / V_{TH}\right)\left(R_2 / R_1\right)$

[5]

b)



The input admittance of the transistor is

$$Y_{in} = \frac{di_{in}}{dv_{in}} = \frac{\left(s\left(C_{BE} + (1-G)C_{CE}\right) + G_\pi + g_m\right) dV_{BE}}{V_{BE}} \simeq sC_{BE} + g_m$$

The amplifier input admittance is $Y_{amp} = Y_{in} // R_1 = sC_{BE} + g_m + G_1$

[10]

c) The source impedance is Z_T. The output impedance is the same as that of the emitter degenerated CE amplifier. This has negative series-series feedback with loop gain $g_m\left(R_1 // R_T\right)$, and open loop output impedance is R_{CE} . Therefore,

$$Z_{out} = \left(\left(1 + g_m\left(R_1 // Z_T\right)\right) R_{CE}\right) // R_2$$

[5]

d) Like all similar amplifiers, this will be a two-pole transfer function. there will be an input pole at $\tau_{in} = \left(\frac{R_1 + R_T}{R_1 R_T} + g_m\right)^{-1}\left(C_{BE} + (1-G)C_{CE}\right)$, and an output pole at $\tau_{out} \simeq \left(C_{CE} + C_{BC}\right) R_2$

The CE amplifier has an input pole at $\tau_{in} = \left(\frac{R_1 + R_T}{R_1 R_T} + \frac{g_m}{\beta}\right)^{-1}\left(C_{BE} + (1+G)C_{BC}\right)$

Both the resistance and the capacitance are greater in the CE case. It is conceivable that a common base amplifier with high gain can have the input pole can be sent to frequencies higher than the output pole.

[10]

4. [a,b: variation on a taught problem]

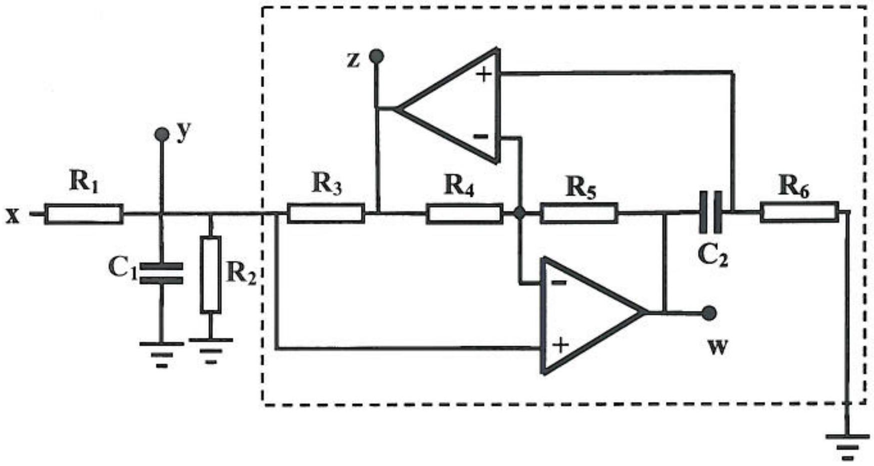


Figure 4.1

a) This is a gyrator, so that $Z_{in} = \frac{R_3 R_5 R_6 s C_2}{R_4}$

(direct citation will get 2/5 marks, since it is in the lecture notes.)
The proof is:

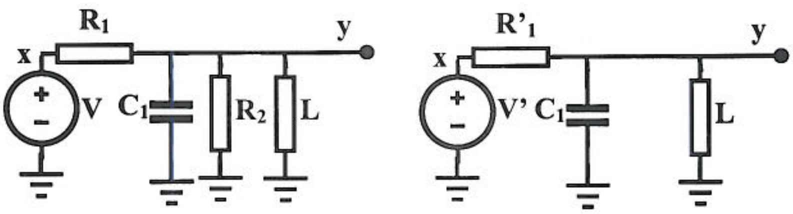
$$V_w = V_y + \frac{V_y}{sC_2R_6}$$

$$V_z = V_y - \left(V_w - V_y\right)\frac{R_4}{R_5} = V_y - \frac{V_y}{sC_2R_6}\frac{R_4}{R_5}$$

$$i_{in} = \left(V_y - V_z\right)/R_2 = \frac{V_y}{sC_2R_6}\frac{R_4}{R_5R_2} \Rightarrow Z_{in} = \frac{sC_2R_6R_5R_2}{R_4} = sL$$

[10]

b) The circuit simplifies to the following RLC circuit:



$$R_1' = R_1 // R_2$$

$$V' = V\frac{R_2}{R_1 + R_2}$$

A Thevenin-Norton transformation gives:

$$V_y = \frac{V'}{R_1'}\left(\frac{1}{R_1'} + sC_1 + \frac{1}{sL}\right)^{-1} = \frac{V'}{R_1'}\frac{R_1' sL}{s^2C_1LR_1' + sL + R_1'} = V'\frac{sL/R_1'}{s^2C_1L + sL/R_1' + 1}$$

This is a band-pass filter with gain $H_0 = V'/V = \frac{R_2}{R_1+R_2}$, $\omega_n = \frac{1}{\sqrt{LC_1}} = \sqrt{\frac{R_4}{C_1C_2R_3R_5R_6}}$

and $Q = \frac{R_1'}{L\omega_n}$

[10]

c) [new problem]

$$V_z = V_y - (V_w - V_y)\frac{R_4}{R_5} = V_y\left(1 - \frac{1}{sC_2R_6}\frac{R_4}{R_5}\right) = V_y\frac{sC_2R_6R_5 - R_4}{sC_2R_6R_5} \Rightarrow$$

$$\frac{V_z}{V_x} = \frac{R_2}{R_1+R_2}\frac{sC_2R_6R_5 - R_4}{sC_2R_6R_5}\frac{sL/R_1'}{s^2C_1L + sL/R_1' + 1} = \frac{R_2}{R_1+R_2}\frac{L}{R'C_2R_6R_5}\frac{sC_2R_6R_5 - R_4}{s^2C_1L + sL/R_1' + 1}$$

This is an inverting lowpass filter, with same natural frequency and quality factor, and gain

$$H_0 = \frac{R_2}{R_1+R_2}\frac{-R_4L}{R'C_2R_6R_5}$$

[5]

d) [new example]

$$V_w = V_y + \frac{V_y}{sC_2R_6} = V_y\left(1 + \frac{1}{sC_2R_6}\right)$$

$$V_z = V_y - (V_w - V_y)\frac{R_4}{R_5} = V_y - \frac{V_y}{sC_2R_6}\frac{R_4}{R_5} = V_y\left(1 - \frac{1}{sC_2R_6}\frac{R_4}{R_5}\right)$$

$\Rightarrow$

$$V_w - V_z = \left(1 + \frac{R_4}{R_5}\right)\frac{1}{sC_2R_6} = LPF$$

A difference amplifier.

[5]