IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE PART II: MEng, BEng and ACGI

Corrected Copy

Q1

# ANALOGUE ELECTRONICS 2

Monday, 14 June 2:00 pm

Time allowed: 2:00 hours

**There are FOUR questions on this paper.**

**Q1 is compulsory.**
**Answer Q1 and any two of questions 2-4.**
**Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).**

**Any special instructions for invigilators and information for candidates are on page 1.**

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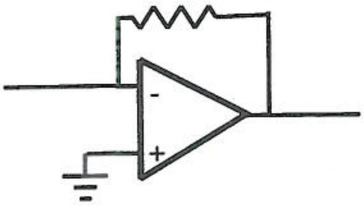
| Examiners responsible | First Marker(s) : | C. Papavassiliou, C. Papavassiliou |
| | Second Marker(s) : | K.D. Harris, K.D. Harris |

## The Questions

1. (Compulsory)

a) Calculate the input impedance of the following circuit. The amplifier has a voltage gain of G=15 and is otherwise ideal. The resistor has a value of 32 $k\Omega$. The op-amp gain does not depend on frequency.




[4]

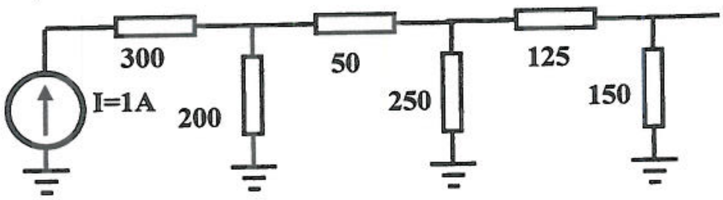
b) Draw a small signal equivalent circuit for a MOSFET which has the body terminal connected to the source. How does this model differ from the small signal model of a bipolar transistor?

[4]

c) Explain what is a dominant pole amplifier. Write an expression for the open loop gain as a function of frequency of a dominant pole amplifier.

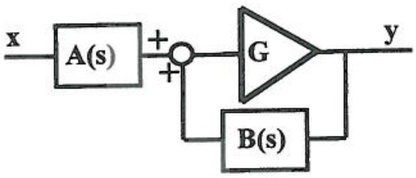
[4]

d) Calculate the Thevenin equivalent of the following network: (the components are resistors)




[4]

e) Calculate the transfer function of the filter in the figure below, if both $A(s)$ and $B(s)$ are first order low pass filters with poles at $\omega_A$ and $\omega_B$ respectively. The amplifier gain G is constant with frequency. What type of filter is this? What is the maximum value of G allowed, and why?




[4]

f) What kind of negative feedback connection can be used to decrease the input impedance and increase the output impedance of an amplifier by a factor of X? What is the character of this amplifier (voltage, current, transimpedance or transconductance)? Will the gain increase or decrease with this feedback connection? By what factor will the gain change?

[4]

g) Write an expression for the transfer function of a second order bandpass filter of peak gain G = 10, quality factor Q=2 and natural frequency $f_0 = 1\,\text{kHz}$

[4]

h) A filter has the following transfer function. State the function and calculate the centre frequency, maximum gain and quality factor of this filter.

$$G(f) = \frac{3 - 3 \cdot 10^{-8} f^2}{10^{-8} f^2 - 2 \cdot 10^{-5} jf - 1}$$

[4]

i) An op-amp is specified to have a gain-bandwidth product of 1 MHz. This op-amp is used to construct an inverting amplifier. Calculate the maximum gain that can be achieved at a frequency of 100 kHz.

[4]

j) Which of the three single transistor amplifiers is most suitable for obtaining a large voltage gain at high frequencies? Explain your answer.

[4]

2. For the filter in figure 2.1:

a) Assume that the op-amp is ideal. Calculate the transfer function of this filter. [10]

b) Identify the type of the filter. Sketch the magnitude and phase Bode plots for this filter. [5]

c) Assume that the op-amp has a finite gain G. Calculate the transfer function of this filter. Does the filter still perform the function you identified in part (b)? What has changed? [10]

d) Assume that the op-amp is a dominant pole amplifier with DC gain $G_0$ and dominant pole at $\omega_0$. Calculate the transfer function of the filter. Estimate the maximum value you can give the RC product in terms of $G_0$ and $\omega_0$ without significantly losing the functionality you identified in parts (a) and (b). [5]

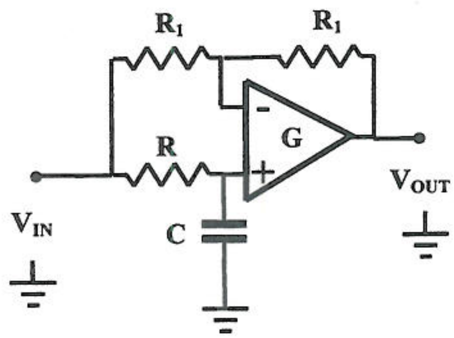



Figure 2.1: Circuit for Question 2.

3. For the filter in figure 3.1, assume that the op-amps are ideal, except in part (e).

a) Calculate the transfer function of the circuit in the dashed box, i.e the relation between $V_{OUT}$ and $V_1$.

[5]

b) Derive an expression for the input admittance of the circuit in the dashed box. Show that this circuit behaves like a large grounded capacitor.

[10]

c) Draw a simplified equivalent circuit for the filter in figure 3.1. Calculate its transfer function. What function does the filter perform? Calculate the pole frequencies and gain of this filter if :

$$C = 1pF\ ,\ R = R_2 = R_3 = 10k\Omega\ ,\ R_1 = R_4 = 10\Omega$$

[5]

d) Show that if C and $R_2$ are exchanged the circuit in the dashed box behaves like a grounded inductor. What are the limitations of such an active inductor implementation?

HINT: in an actual application the op-amp will have a finite gain-bandwidth product.

[5]

e) Prove that if C and $R_3$ are exchanged the circuit in the dashed box behaves like a grounded inductor. Why is this implementation of an inductor unlikely to work?

[5]

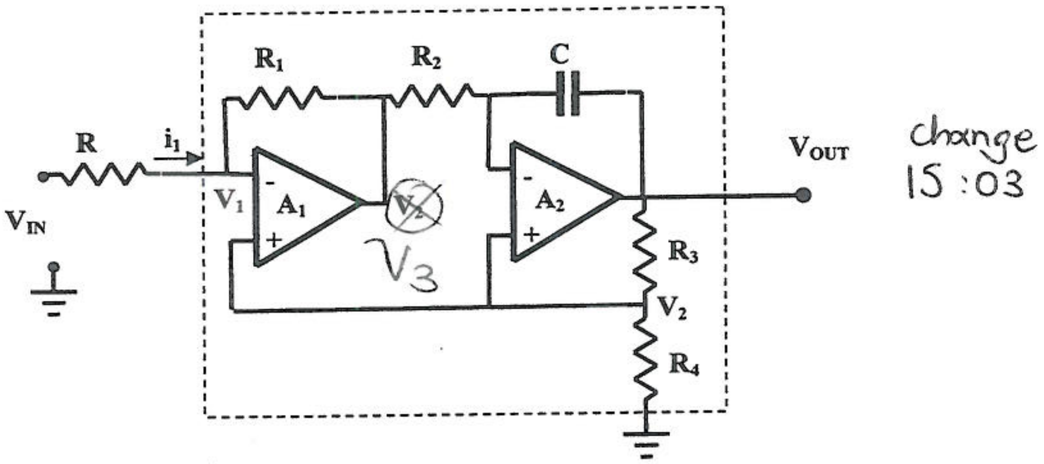



Figure 3.1: Circuit for Question 3.

4. Consider the emitter follower amplifier shown in figure 4.1, together with a signal source and a load. You may assume that the capacitor is infinitely large.

The bipolar transistor Q has $f_T = 10\,\text{MHz}$, $V_A = 50\,\text{V}$. Its DC current gain is $\beta_0 = 100$

The current source has a magnitude $I_0 = 100\,\text{mA}$ and the power supply is $V_{CC} = 10\,\text{V}$.

a) Identify the role of each of the dashed boxes in figure 4.1. What is the role of the current source?

[5]

b) Draw a small signal equivalent circuit for the transistor. Calculate values for all elements on the equivalent circuit except $R_{BB}$, $C_{BC}$ and $C_{CE}$ which you may assume are negligible.

[5]

c) Write an expression for the current gain of the transistor as a function of frequency.

[5]

d) Using the Miller theorem, or otherwise, derive an expression for the frequency dependence of the input impedance of the amplifier. Assume that $Z_L$ is real, and $\beta_0 Z_L >> R_\pi$.

[5]

e) Assume that $Z_L$ is a capacitor. Derive an expression for the input impedance of the emitter follower amplifier. Show that in this case the input impedance has a negative real part. Evaluate the DC limit of the input impedance of the capacitively loaded emitter follower. What is the minimum value the source resistance $R_S$ can take?

[5]

f) Identify the type of feedback present in this amplifier. Assume the source is resistive and the load a capacitor. Derive an expression for the voltage gain $G(s) = \frac{V_L}{V_S}$ as a function of frequency. Comment on your result.

[5]

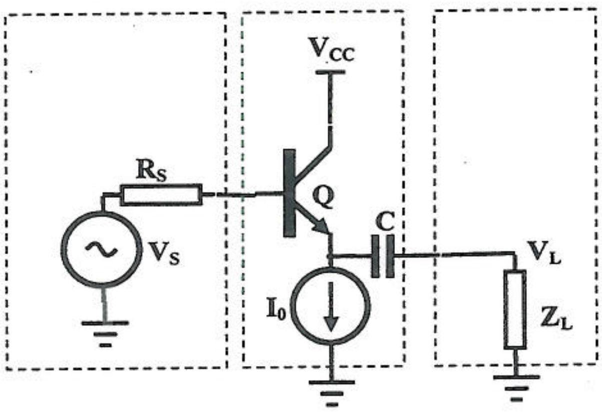



Figure 4.1: Circuit for Question 4.

Master – May '10

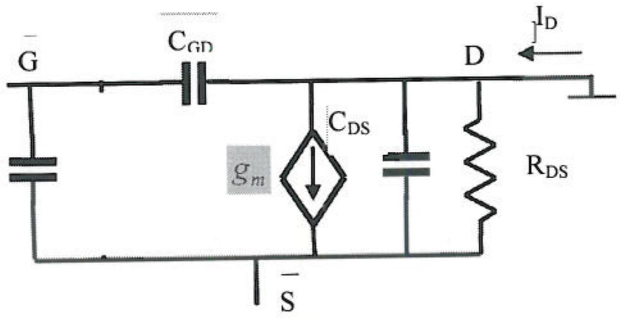
E2.2

## The Answers 2010

ANSWER QUESTION 1: (4 marks each) ([B] bookwork, [C] computed example)

a) [C] 2kOhms by application of the Miller theorem.

b) [B] The model is the same as of a BJT without R-pi




c) [B] One pole at a much lower frequency than any other poles – zeroes in the response, or rather, when other characteristic frequencies are at frequencies higher than the unity gain frequency. The gain dependence on frequency is approximately:

$$G(s)=\frac{G_{DC}}{1+s\tau}$$

d) [C] The 300 Ohm resistor is irrelevant. Step by step:

$V_T = 200V \quad , R_T = 200\,\Omega$

$V_T = 100V \quad , R_T = 125\,\Omega$

$V_T = 37.5V \quad , R_T = 93.75\,\Omega$

e) [C]

$$H(s)=\frac{AG}{1-BG}=\frac{\frac{G}{1+s\tau_A}}{1-\frac{G}{1+s\tau_B}}=\frac{G(1+s\tau_B)}{(1+s\tau_B)(1+s\tau_A)-G(1+s\tau_A)}=$$

$$=\frac{G(1+s\tau_B)}{s^2\tau_A\tau_B+s(\tau_A+\tau_B-G\tau_A)+(1-G)}=\frac{G}{1-G}\frac{1+s\tau_B}{s^2/\omega_0^2+2\zeta s/\omega_0+1}$$

This is a low pass filter and G cannot be greater than 1, or Q will be negative

f) [B] Shunt-series negative feedback. The gain (current gain!) will reduce by a factor X.

g) [C] let $\omega_0 = 2\pi\ kHz$ then $G(s)=\dfrac{10}{-\omega^2/\omega_0^2+j\omega/2\omega_0+1}$

h) [C] It is a band stop filter centred at $10^4$ Hz, peak gain of -3 and Q=5

i) [B] The maximum possible gain is $10/\sqrt{2}$ (I will accept 10 as a correct answer)

j) [B] Common base . Because it has unity current gain and positive Miller feedback.

ANSWER QUESTION 2: [New exercise]

a) If the op-amp is ideal, then

$$V_+ = V_- \Rightarrow (V_{in} + V_{out})/2 = \frac{V_{in}}{1+sRC} \Rightarrow V_{out} = V_{in}\left(\frac{2}{1+sRC} - 1\right) \Rightarrow$$

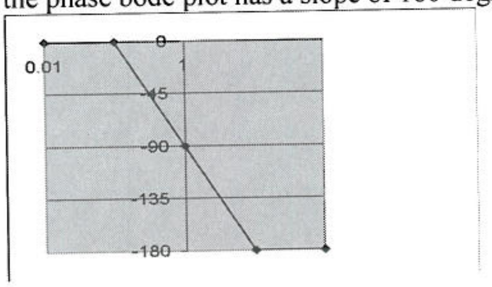
$$\frac{V_{out}}{V_{in}} = \frac{1-sRC}{1+sRC}$$

[10]

b) This is a first order all pass filter

The magnitude Bode plot is trivial (constant with frequency)

the phase bode plot has a slope of 180 degrees per decade




[5]

c) If the amplifier has a finite gain G then

$$V_{out} = G(v_+ - v_-) = G\left(\frac{V_{in}}{1+sRC} - (V_{in} + V_{out})/2\right) \Rightarrow$$

$$(2+G)V_{out} = GV_{in}\frac{1-sRC}{1+sRC} \Rightarrow V_{out} = \frac{G}{2+G}V_{in}\frac{1-sRC}{1+sRC}$$

It performs, therefore, the same function but at a smaller amplitude.

[10]

d)

$$\frac{V_{out}}{V_{in}} = \frac{G}{2+G}\frac{1-sRC}{1+sRC} = \frac{G_0}{2+G_0+2s\tau_0}\frac{1-sRC}{1+sRC} = \frac{G_0}{2+G_0}\frac{1}{1+s\tau'}\frac{1-sRC}{1+sRC}$$

$$\tau' = \frac{2\tau_0}{2+G_0}$$

as long as $\tau' << RC$ the function of the filter is not significantly altered.

[5]

ANSWER Question 3. [New exercise]

a) Let $H = \frac{R_4}{R_3 + R_4}$, By the golden rule applied on $A_1$ , the box gain is 1/H !

[5]

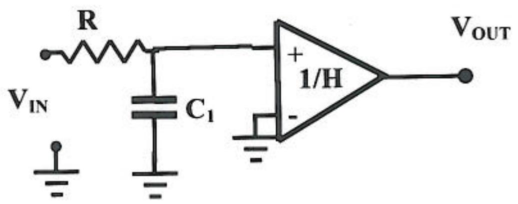
b) $i_{in} = G_1(V_1 - V_2), V_1 = HV_{out}$ but

$$(V_2 - HV_{out})G_2 = (H-1)V_{out}sC \Rightarrow$$
$$V_2 = V_{out}\left[(H-1)sR_2C + H\right] = V_{in}\left[(1-1/H)sR_2C + 1\right] \Rightarrow$$
$$i_{in} = G_1(V_1 - V_2) = G_1\left(1 - \left[(1-1/H)sR_2C + 1\right]\right) \Rightarrow$$
$$Y_{in} = i_{in}/V_1 = G_1\left((1-H)/H\right)sR_2C = sC\frac{R_2}{R_1}\frac{R_3}{R_4}$$

Therefore C can be amplified by the ratio of the resistor products.

[10]

c)




This is an RC low pass filter with

$C_1 = C\frac{R_2}{R_2}\frac{R_3}{R_4}$ followed by a gain 1/H (H<1 since it is a voltage divider!)

The pole is at $\tau = RC\frac{R_2R_3}{R_1R_4}$. With the numbers given, H=$10^{-3}$, $\tau = 10^{-2}\sec^{-1} \Rightarrow f_p = 6Hz$ (!)

d) The dashed circuit is really a GIC. Therefore $Y = Y_F\frac{R_2}{R_1}\frac{R_3}{R_4}$, an inductor can be emulated by making $Y_F$ a resistor and $R_2$ or $R_3$ a capacitor.

If a capacitor is used in the place of $R_2$ there is an embedded "ideal" differentiator in the circuit which will ring if the op-amp has a finite gain bandwidth product. (note that the students are not required to know what a GIC is, only to observe they can give this admittance inductive character by properly placing the capacitor)

[5]

e) If a capacitor is used in the place of $R_3$ the positive feedback path gain around $A_2$ will be increasing with frequency and will cause $A_2$ to oscillate.

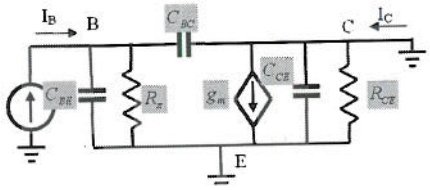
[5]

ANSWER Question 4. Each part is 5 marks. [a,b anc c are bookwork. The rest is new computed example]

a) From left to right source, amplifier, load. The current source establishes the operating point.

b) The model is the usual $\pi$ model,




With

$$g_m = \frac{100mA}{25mV} = 4S, R_{CE} = \frac{50V}{100mA} = 0.5k\Omega, R_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\Omega,$$

$$\frac{g_m}{2\pi C_{BE}} = f_T = 10\,\text{MHz} \Rightarrow C_{BE} = \frac{g_m}{2\pi f_T} = 63.7nF$$

c) The current gain of the transistor is:

$$h_{fe}(s) = \frac{\beta_0}{1 + j\beta_0 f / f_T}$$

d) By application of the Miller theorem, the input impedance is

$$Z_{in} = R_\pi // C_{BE} + (h_{fe} + 1) Z_L = \frac{R_\pi}{1 + sC_{BE}R_\pi} + \left(\frac{\beta_0}{1 + s\beta_0 / \omega_T} + 1\right) Z_L =$$

$$= \frac{R_\pi}{1 + s\beta_0 / \omega_T} + \left(\frac{\beta_0 + 1 + s\beta_0 / \omega_T}{1 + s\beta_0 / \omega_T}\right) Z_L \simeq Z_L(\beta_0 + 1)\frac{1 + s\beta_0 / (\beta_0 + 1)\omega_T}{1 + s\beta_0 / \omega_T} \simeq \frac{Z_L(\beta_0 + 1)}{1 + s\beta_0 / \omega_T}$$

e) if $Z_L = 1/sC$ then

$$Z_{in} \simeq \frac{1}{sC}\frac{\beta_0 + 1}{1 + s\beta_0 / \omega_T} = \frac{\beta_0 + 1}{j\omega C - \omega^2 \beta_0 C / \omega_T} \Rightarrow \operatorname{Re} Z_{in} = \frac{-\omega^2 \beta_0 / \omega_T C(\beta_0 + 1)}{\omega^2 C^2 + \omega^4 \beta_0^2 C^2 / \omega_T^2}$$

$$\lim_{\omega \to 0} \operatorname{Re} Z_{in} = \frac{-\beta_0}{C\omega_T(\beta_0 + 1)} \simeq \frac{-1}{C\omega_T}\left( = \frac{-C_{BE}}{g_m C}\right)$$

The source resistance must be bigger than this value.

f) This is series-series feedback. The voltage gain of the amplifier is

$G = \frac{g_m Z_L}{1 + g_m Z_L}$. The source divider is $G_S = \frac{Z_{in}}{Z_S + Z_{In}}$ so that the overall gain is

$$G = \frac{g_m Z_L}{1 + g_m Z_L} \frac{Z_{in}}{R_S + Z_{in}} = \frac{g_m}{sC + g_m} \frac{\beta_0 + 1}{R_S \left( sC + s^2 \beta_0 C / \omega_T \right) + \beta_0 + 1} =$$

$$= \frac{g_m}{sC + g_m} \frac{1}{s^2 R_S \beta_0 C / \omega_T \left( \beta_0 + 1 \right) + R_S sC / \left( \beta_0 + 1 \right) + 1}$$

This is an LPF with $\omega_0 = \sqrt{\omega_T \left( \beta_0 + 1 \right) / \beta C R_S} \simeq \sqrt{\omega_T / C R_S}$ and $Q = \left( \beta_0 + 1 \right) / \sqrt{\omega_T C R_S}$

For the emitter follower to be overdamped, the condition on $R_S$ is further restricted to

$R_S > \left( \beta + 1 \right)^2 / \omega_T C$