

Paper Number(s): **E1.3**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE  
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2002

EEE PART I: M.Eng., B.Eng. and ACGI

*Modified formula sheet  
(0-30.)*

**DEVICES AND FIELDS**

Monday, 10 June 10:00 am

There are FIVE questions on this paper.

There are two sections. Answer THREE questions including at least ONE question from each section.

Use a separate answer book for each section.

**Corrected Copy**

Time allowed: 2:00 hours

**Examiners responsible:**

First Marker(s): Fobelets, K. Popovic, D.

Second Marker(s): Leaver, K.D. Pal, B.C.

SECTION A

Answer at least ONE question. Use a separate answer book for each section.

Formulae and Constants

$$I_{DS} = \frac{WC_{ox}\mu}{L} \left[ (V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$g_m = \frac{dI_{DS}}{dV_{GS}}$$

$$f_c = \frac{g_m}{2\pi C_{GS}}$$

$$J = \left( \frac{qD_h p_{n_0}}{L_n} + \frac{qD_e n_{p_0}}{L_p} \right) \left( \exp\left(\frac{qV}{kT}\right) - 1 \right)$$

$$V_o = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$W_n = \sqrt{\frac{2\epsilon_0 \epsilon_r}{q} (V_o - V) \frac{N_A}{N_A N_D + N_D^2}}$$

$$W_p = \sqrt{\frac{2\epsilon_0 \epsilon_r}{q} (V_o - V) \frac{N_D}{N_A N_D + N_A^2}}$$

$$n = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$p = N_V \exp\left[\frac{(E_V - E_F)}{kT}\right]$$

$$\sigma = q[\text{carrier concentration}]\mu$$

$$D = \frac{\mu kT}{q}$$

$$q = 1.602 \times 10^{-19} \text{ C}$$

$$n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ for Si at 300K}$$

$$kT/q = 0.026 \text{ V at 300 K}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$N_{CSi} = 2.8 \times 10^{19} \text{ cm}^{-3}$$

$$N_{VSi} = 1.04 \times 10^{19} \text{ cm}^{-3}$$

Relative permittivity of SiO<sub>2</sub> = 4

Relative permittivity of Si = 11.7

$$\left. \begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} \text{ F m}^{-1} \\ \mu_0 &= 4\pi \times 10^{-7} \text{ H m}^{-1} \end{aligned} \right\}$$

written on board of 10-30

# 1 MOSFET

Questions (a) – (e) concern an n-channel enhancement mode MOSFET.

a) What is the majority carrier type in this device? [1]

b) Give a definition for the threshold voltage. [2]

c) Is the threshold voltage of this device positive or negative? [1]

d) Draw the energy band diagram, including  $E_c, E_v, E_F$  and  $E_g$  of the n-channel enhancement mode MOSFET from gate to substrate (body) when the channel is conducting (inversion). The position of the Fermi level relative to a reference level before the materials are in contact is given in Figure 1. [6]

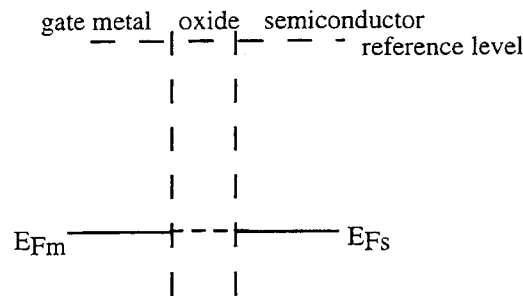


Figure 1: The position of the Fermi level in metal, oxide and semiconductor before contact.

e). Reducing the size of MOS transistors is a way to improve their transconductance  $g_m$ . Derive an expression for the ratio of the transconductances of the enhancement mode MOSFET in *saturation* when both the gate length and the oxide thickness are changed from:  $L_1$  to  $L_2$  and  $t_{ox1}$  to  $t_{ox2}$  [10]  
If both are halved, what will this ratio be?

## 2 Junctions

### a) Metal-semiconductor contact

Given the energy band diagram of a metal and semiconductor before contact (see Figure 2), sketch the energy band diagram, including  $E_c, E_v, E_F$  and  $E_g$  of the metal-semiconductor junction in contact. [4]

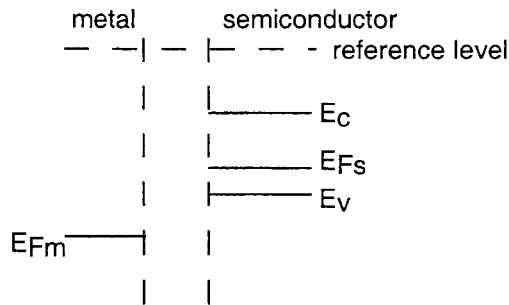


Figure 2: Energy band diagrams of a metal and semiconductor before contact

### b) pn junction

Given the energy band diagram of two heavily doped semiconductors before contact (see Figure 3), sketch the energy band diagram of the same semiconductors in contact. Note that although the Fermi levels are lying outside the energy gap all the usual rules are followed. [4]

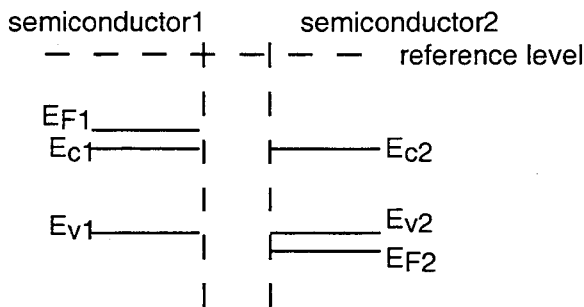


Figure 3: Energy band diagrams of two heavily doped semiconductor before contact

### c) Is the following statement correct or wrong?

The hole diffusion current of a pn junction under zero bias is zero.

Give a brief reason for your answer. [4]

d) Calculate the junction capacitance of the following Si pn diode under 1V reverse bias. The diode area is  $10 \mu\text{m} \times 10 \mu\text{m}$ , the donor concentration is  $10^{16} \text{ cm}^{-3}$ , the acceptor concentration is  $10^{19} \text{ cm}^{-3}$ . The temperature  $T=300\text{K}$ . Justify any approximations you make. [8]

### 3 Bipolar Junction Transistor

a) State whether the main current flowing from emitter to collector through the base region of a BJT is governed by drift or diffusion. Give a brief reason for your answer. [3]

b) The base current,  $I_B$ , of a BJT consists of three components such that  $I_B = I_{B'} + I_{B''} + I_{CB0}$  [3]  
Give a short description of the three physical processes which cause these three current components. Remember the biasing configuration of the active device given in Figure 4.

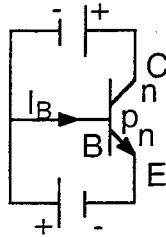


Figure 4: biasing configuration of npn transistor

c) Give the expression for the small signal current gain  $\beta$  of the BJT in terms of the external voltages and currents. [4]

Give two ways in which to improve the current gain.

d) Calculate the current density injected from the emitter into the base of the following Si npn transistor with emitter doping of  $10^{19} \text{ cm}^{-3}$ , base doping of  $10^{16} \text{ cm}^{-3}$  and collector doping  $10^{16} \text{ cm}^{-3}$ . Bias voltages are  $V_{EB} = 0.7\text{V}$  forward bias and  $V_{BC}$  is 3V reverse bias, emitter and collector length :  $L_n = 100 \mu\text{m}$ , and effective base width  $L_p = 50 \mu\text{m}$ . Mobility of emitter electrons is  $100 \text{ cm}^2/\text{Vs}$ , of holes is  $200 \text{ cm}^2/\text{Vs}$  and of collector electrons is  $300 \text{ cm}^2/\text{Vs}$ . The device is operating at room temperature  $T = 300\text{K}$ . [10]

## SECTION B

Answer at least ONE question. Use a separate answer book for each section

4. (a) Explain Gauss' Law. Under what circumstances is it useful? [5marks]
- (b) Consider the three closed surfaces,  $S_1$ ,  $S_2$  and  $S_3$ , in the field of four point charges as shown in Figure 1. What is the flux of the electric field intensity vector through each of these surfaces? Assume that all four point charges are situated in a vacuum. [5marks]
- (c) What is the absolute potential at a point P which is 2m from a point charge  $Q = +5\mu\text{C}$ ? What is the work required to move a  $+5\text{nC}$  charge from infinity to point P? Assume a relative permittivity  $\epsilon_r = 1$ . [10marks]

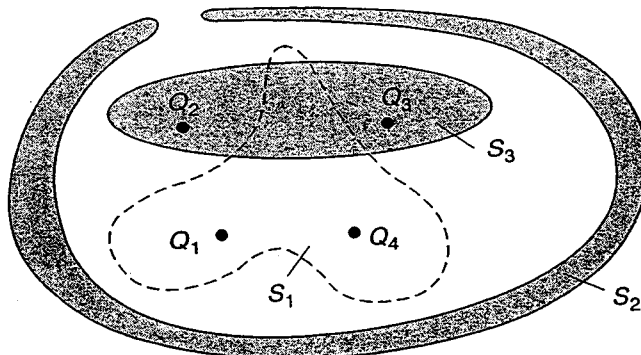


Figure 1

5. (a) What is a fundamental difference between electric field lines and magnetic field lines? [5marks]

(b) Current  $I$  flows in the inner conductor of a long coaxial cable and returns through the outer conductor. What is the magnetic field in the region outside the coaxial cable and why? [5marks]

(c) The rectangular loop shown in Figure 2 is situated in the  $x - y$  plane and moves away from the origin at a velocity  $\mathbf{u} = \hat{y} 5$  (m/s) in a magnetic field given by

$$\mathbf{B}(y) = \hat{z} 0.2e^{-0.1y} \quad (\text{T})$$

If  $R = 5\Omega$ , find the current  $I$  at the instant that the loop sides are at  $y_1 = 2\text{m}$  and  $y_2 = 2.5\text{m}$ . The loop resistance may be ignored. [10marks]

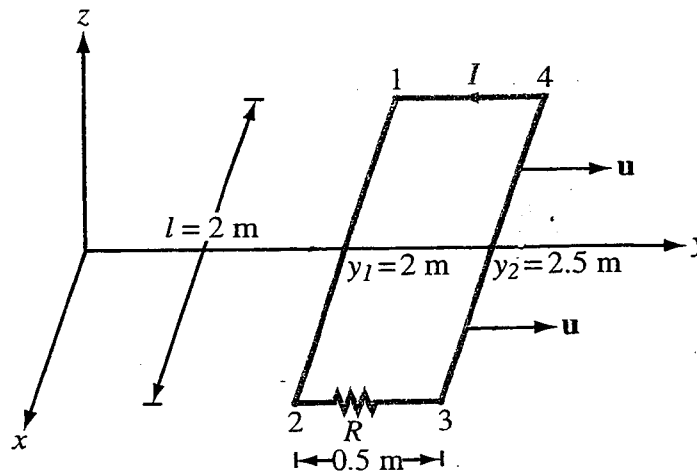


Figure 2

**1 MOSFET**

a) electrons

[1]

b) either of the following answers is acceptable:

[2]

i) the threshold voltage is the *gate – source voltage* at which the *channel* is *created* in an enhancement mode MOSFET or *depleted* in a depletion mode MOSFET

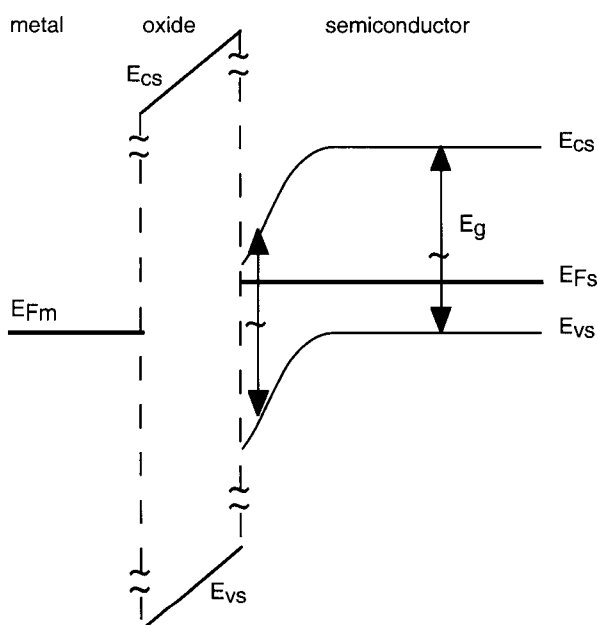
ii) is the *gate – source voltage* at which the *inverted carrier concentration* at the SiO<sub>2</sub>-Si interface is the same as the majority carrier concentration in the bulk far away from the junction where no band bending occurs

ii) is the *gate – source voltage* at which the MOS capacitance is *minimum*

c) positive

[1]

d)



[6]

e)

$$I_{DS} = \frac{WC_{ox}\mu}{L} \left[ (V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right]$$

in saturation :

$$V_{DS} = (V_{GS} - V_T)$$

$$I_{DS}^{sat} = \frac{WC_{ox}\mu}{2L} (V_{GS} - V_T)^2$$

transconductance :

$$g_{m1}^{sat} = \frac{dI_{DS}^{sat}}{dV_{GS}}$$

$$g_{m1}^{sat} = \frac{WC_{ox}\mu}{L} (V_{GS} - V_T)$$

oxide capacitance :

$$C_{ox} = \frac{\epsilon_o \epsilon_{ox}}{t_{ox}}$$

$$g_{m1}^{sat} = \frac{W\epsilon_o \epsilon_{ox}\mu}{t_{ox1}L_1} (V_{GS} - V_T) \quad \& \quad g_{m2}^{sat} = \frac{W\epsilon_o \epsilon_{ox}\mu}{t_{ox2}L_2} (V_{GS} - V_T)$$

transconductance ratio :

$$\frac{g_{m2}^{sat}}{g_{m1}^{sat}} = \frac{t_{ox1}L_1}{t_{ox2}L_2}$$

Transconductance improvement when :

$$t_{ox2} = t_{ox1}/2 \quad \& \quad L_2 = L_1/2$$

$$\frac{g_{m2}^{sat}}{g_{m1}^{sat}} = 4$$

[10]

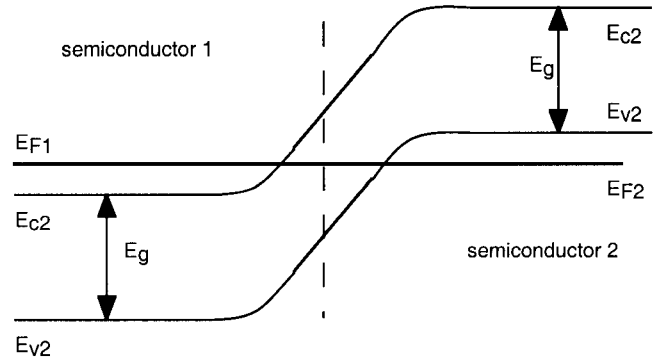
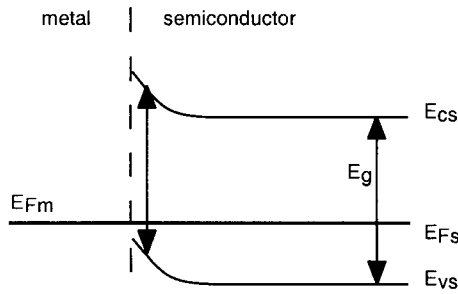
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**2 Junctions**

a) metal-semiconductor contact

b) pn junction



[4]

[4]

c) Correct or wrong  
wrong

[1]

At zero bias there exists a drift current due to the built-in voltage and therefore a diffusion component has to exist to cancel the drift current and give zero net hole current.

[3]

d) pn diode capacitance in reverse bias is due to the depletion width W.

The depletion width extends in the lowest doped region. Since the doping density in the n-type region is 1000 times smaller than in the p-type region, we can neglect the contribution of the depletion width in the highest doped region.

Depletion width extends into the n - doped region :

$$W \approx W_n = \sqrt{\frac{2\epsilon_0\epsilon_r(V_0 - V)}{q} \frac{N_A}{N_D(N_A + N_D)}}$$

$$N_A \gg N_D$$

$$W \approx W_n \approx \sqrt{\frac{2\epsilon_0\epsilon_r(V_0 - V)}{q} \frac{1}{N_D}}$$

$$\text{built - in voltage : } V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$V_0 = 0.026V \ln \frac{10^{16} 10^{19} \text{ cm}^{-6}}{2.1025 \cdot 10^{20} \text{ cm}^{-6}} = 0.879V$$

$$W \approx \sqrt{\frac{2 \cdot 8.854 \cdot 10^{-14} \text{ F/cm} \cdot 11.7}{1.602 \cdot 10^{-19} \text{ C}} (0.879V + 1V) \frac{1}{10^{16} \text{ cm}^{-3}}}$$

$$W \approx 49.297 \cdot 10^{-6} \text{ cm}$$

$$\text{Depletion capacitance : } C = \frac{\epsilon_0\epsilon_r}{W} A$$

$$A = 10\mu\text{m} \times 10\mu\text{m} = 10 \cdot 10^{-4} \text{ cm} \times 10 \cdot 10^{-4} \text{ cm} = 10^{-6} \text{ cm}^2$$

$$C \approx \frac{8.854 \cdot 10^{-14} \text{ F/cm} \cdot 11.7}{49.297 \cdot 10^{-6} \text{ cm}} 10^{-6} \text{ cm}^2$$

$$C \approx 2.1 \cdot 10^{-14} \text{ F}$$

Without approximation :

$$W_n = \sqrt{\frac{2\epsilon_0\epsilon_r(V_0 - V)}{qN_D} \frac{(N_A)}{(N_A + N_D)}}$$

$$W_n = \sqrt{\frac{2 \cdot 8.854 \cdot 10^{-14} \text{ F/cm} \cdot 11.7 (0.879V + 1V)}{1.602 \cdot 10^{-19} \text{ C}} \frac{10^{19} \text{ cm}^{-3}}{(10^{16})(10^{19} + 10^{16}) \text{ cm}^{-3}}}$$

$$W_n = 49.27 \cdot 10^{-6} \text{ cm}$$

$$W_p = \sqrt{\frac{2\epsilon_0\epsilon_r(V_0 - V)}{qN_A} \frac{(N_D)}{(N_A + N_D)}}$$

$$W_p = \sqrt{\frac{2 \cdot 8.854 \cdot 10^{-14} \text{ F/cm} \cdot 11.7 (0.879V + 1V)}{1.602 \cdot 10^{-19} \text{ C}} \frac{10^{16} \text{ cm}^{-3}}{(10^{19})(10^{19} + 10^{16}) \text{ cm}^{-3}}}$$

$$W_p = 0.04927 \cdot 10^{-6} \text{ cm}$$

$$W = W_n + W_p = 49.32 \cdot 10^{-6} \text{ cm}$$

$$C = \frac{8.854 \cdot 10^{-14} \text{ F/cm} \cdot 11.7}{49.32 \cdot 10^{-6} \text{ cm}} 10^{-6} \text{ cm}^2$$

$$C = 2.1 \cdot 10^{-14} \text{ F}$$

[10]

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Mark allocation in right margin

**3 Bipolar Junction Transistor**

a) diffusion

In the base of the transistor a carrier gradient is generated due to the different signs of the external bias voltages across both base junctions (BE and BC). This concentration gradient in the base results in a diffusion current. [3]

b)  $I_B = I_B' + I_B'' + I_{CB0}$  $I_B'$  is the base current injected into the emitter $I_B''$  is the base recombination current $I_{CB0}$  is the reverse bias leakage current into collector [3]c) The current  $\beta = \frac{dI_C}{dI_B}$  gain:

For high gain: small base width and large emitter doping compared to the base doping [4]

d) Current injected into the base is given by the pn diode current for a n<sup>+</sup>p (E-B) junction (therefore the electron current is much larger than the hole current which can be neglected). [10]

Current density:

$$J = \left( \frac{qD_h p_{n0}}{L_n} + \frac{qD_e n_{p0}}{L_p} \right) \left( \exp\left(\frac{qV}{kT}\right) - 1 \right)$$

$$J = kT \left( \frac{\mu_h p_{n0}}{L_n} + \frac{\mu_e n_{p0}}{L_p} \right) \left( \exp\left(\frac{qV}{kT}\right) - 1 \right)$$

minority carrier concentration:

$$p_p = N_A \rightarrow n_p = \frac{n_i^2}{p_p} = \frac{n_i^2}{N_A} = \frac{(1.45 \cdot 10^{10} \text{ cm}^{-3})^2}{10^{16} \text{ cm}^{-3}} = 21025 \text{ cm}^{-3}$$

$$n_n = N_D \rightarrow p_n = \frac{n_i^2}{n_n} = \frac{n_i^2}{N_D} = \frac{(1.45 \cdot 10^{10} \text{ cm}^{-3})^2}{10^{19} \text{ cm}^{-3}} = 21.025 \text{ cm}^{-3}$$

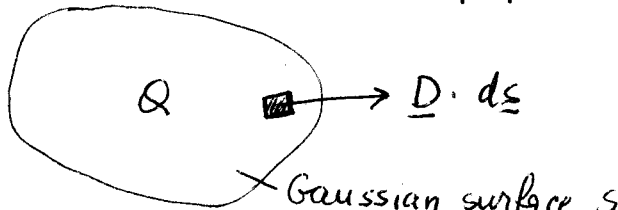
$$J = 0.026 \text{ V} \cdot 1.602 \cdot 10^{-19} \text{ C} \left( \frac{200 \text{ cm}^2/\text{Vs} \cdot 21.025 \text{ cm}^{-3}}{100 \cdot 10^{-4} \text{ cm}} + \frac{100 \text{ cm}^2/\text{Vs} \cdot 21025 \text{ cm}^{-3}}{50 \cdot 10^{-4} \text{ cm}} \right) \left( \exp\left(\frac{0.7 \text{ V}}{0.026 \text{ V}}\right) - 1 \right)$$

$$J = 0.864 \text{ C/cm}^2 \text{ s} = 0.864 \text{ A cm}^{-2}$$

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4. a)  $\oint_S \underline{D} \cdot d\underline{s} = Q$  The outward flux of  $\underline{D}$  through a surface is proportional to the enclosed charge  $Q$ .



It is a convenient method for determining  $\underline{D}$  and/or  $\underline{E}$  when the charge distribution possesses symmetry properties.

(Because at every point on the surface the direction of  $d\underline{s}$  is the outward normal to the surface, only the normal component of  $\underline{D}$  at the surface contributes to the integral above.  $\Rightarrow$  the surface  $S$  should be chosen such that, from symmetry considerations, the magnitude of  $\underline{D}$  is constant and its direction is normal or tangential at every point.)

b) flux through  $S_1 = \frac{Q_1 + Q_4}{\epsilon_0}$

flux through  $S_2 = 0$

flux through  $S_3 = \frac{Q_2 + Q_3}{\epsilon_0}$ .

c)  $V = - \int_{\infty}^r \underline{E} \cdot d\underline{r} = - \frac{Q}{4\pi\epsilon} \int_{\infty}^r \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon r}$  absolute potential

$$V = \frac{5 \cdot 10^{-6}}{4\pi \cdot 8.85 \cdot 10^{-12} \cdot 2} = 22.5 \text{ kV} (= 22.5 \frac{\text{kJ}}{\text{C}})$$

$$W = (22.5 \cdot 10^3) \cdot (5 \cdot 10^{-9}) = 112.5 \cdot 10^{-6} \text{ J}$$

5

5

2

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Question Number etc. in left margin

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5. a) Electric field lines originate from positive electric charges and terminate on negative electric charges. Hence, the electric flux through a closed surface surrounding one of the charges is not zero.

In contrast, magnetic field lines always form continuous closed loops.  $\Rightarrow$  the net magnetic flux through the closed surface ~~is~~ is always zero.  $\oint \underline{B} \cdot d\underline{s} = 0$

- b)  $\underline{H} = 0$  outside the cable because the net current enclosed by the superian contour is zero.

- c)  $\underline{u} \times \underline{B}$  is along  $\hat{x}$   $\Rightarrow$  voltages are induced <sup>across</sup> ~~only~~ the sides oriented along  $\hat{x}$  (ie sides between points 1 and 2 and the side between 3 and 4)

$$\underline{B}(y_1) = \hat{z} 0.2 e^{-0.1y_1} = \hat{z} 0.2 e^{-0.2} \text{ (T)}$$

$$V_{12} = \int_2^1 [\underline{u} \times \underline{B}(y_1)] \cdot d\underline{l} = \int_{l/2}^{-l/2} (\hat{y} 5 \times \hat{z} 0.2 e^{-0.2}) \cdot \hat{x} dx$$

$$= -e^{-0.2} l = -2 e^{-0.2} = -1.637 \text{ volts}$$

Similarly:

$$V_{43} = -u B(y_2) l = -5 \cdot 0.2 e^{-0.25} \cdot 2 = -1.558 \text{ volts}$$

$$I = \frac{V_{43} - V_{12}}{R} = \frac{0.079}{5} = 15.8 \mu\text{A}$$

5

5

4

4

2

20